DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **BM-5**

SEXTUPOLE IN THE “C” CORRECTOR MAGNET

G. Benedetti

Introduction

Measurements of coupling and decoherence show that important sources of linear coupling and non-linearities exist in the Interaction Regions of the DAFNE Main Rings or near them.

Magnetic measurements performed on the “C” correctors showed the existence of sextupole terms in these magnets [1]. The horizontal and vertical “C” correctors are used to adjust the horizontal crossing of the beams and to separate them along the two Interaction Regions.

The coupled Orbit Response Matrices [2] for both the electron and the positron ring can be fitted by a linear skew gradient at the positions of the “C” correctors.

The tune shift observed changing the vertical closed bump around the two Interaction Points is exactly explained by the sextupole terms known from the magnetic measurements [3].

In this note it is shown the origin of the coupling term and one example of the agreement between the measured and calculated dependence of the tunes for the electron ring.

1. Review of the magnetic measurement [1]

Horizontal and vertical scans of the magnetic field measurements performed on the horizontal (CH) and vertical (CV) “C” corrector are shown in Figs. 1 and 2.

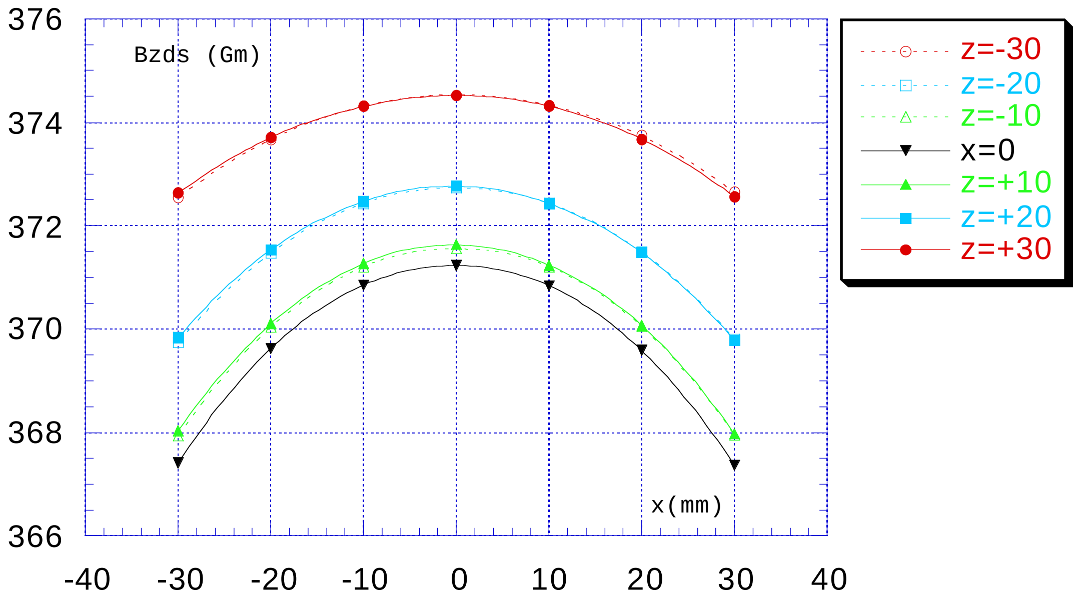

Figure 1 - Horizontal scans of the integrated vertical component of the field along the “C” magnet at different vertical positions (*z* indicates the vertical coordinate that in the text is indicated with *y*) with CH @ 171 A, CV off (from [1]).

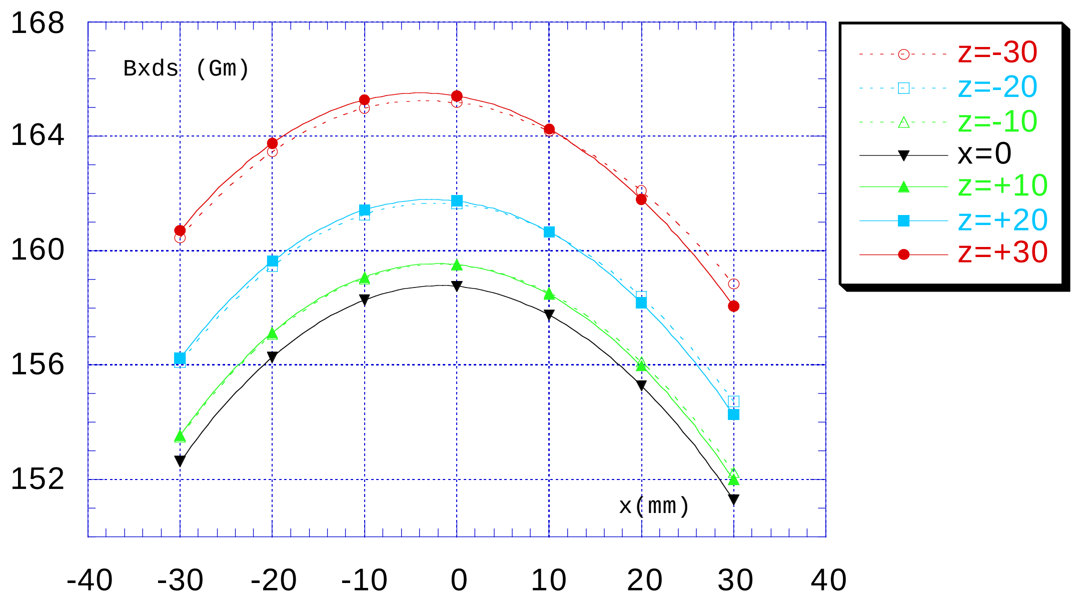

Figure 2 - Horizontal scans of the integrated horizontal component of the field along the “C” magnet at different vertical positions (*z* indicates the vertical coordinate that in the text is indicated with *y*) with CV @ 109 A, CH off (from [1]).

By fitting the curves in Figures 1,2 with a polynomial, the integrated sextupoles terms were calculated. The horizontal corrector has an integrated *normal* sextupole gradient:

$$S_{CH}(T/m) = \int (\partial^2 B_y \big/ \partial x^2) dz = 4.63 \cdot 10^{-3} \cdot I(A) \tag{1.1}$$

while the vertical corrector has an integrated *skew* sextupole gradient:

$$S_{CV}(T/m) = \int (\partial^2 B_x \big/ \partial y^2) dz = 1.35 \cdot 10^{-2} \cdot I(A) \tag{1.2}$$

We can compare the order of magnitude of the integrated sextupole gradient of a C vertical corrector powered at a typical current (I=10÷50 A) with the integrated sextupolar gradients of the sextupoles installed in the DAFNE Main Rings [4]:

“C” CV @ 40 A --> 0.54 T/m

Large Sextupole @ 10 A --> 1.9 T/m

Therefore the “C” sextupole is not negligible when compared with the sextupoles used for chromaticity correction.

2. The sextupolar terms in the “C” Magnet Corrector

The effect of the “C” sextupole depends of course also on the beta functions at its position and its linear contribution depends on the displacement of the beam trajectory from the magnetic center, as explained below.

The horizontal corrector CH has a *normal* sextupolar term in the magnetic field:

$$\begin{aligned} B_x &= -s_{CH} xy \\ B_y &= -\tfrac{1}{2} s_{CH} (x^2 - y^2) \end{aligned} \tag{2.1}$$

that gives the *normal* quadrupolar gradients:

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = -s_{CH} x \tag{2.2}$$

and the *skew* quadrupolar gradients:

$$\frac{\partial B_y}{\partial y} = -\frac{\partial B_x}{\partial x} = s_{CH} y. \tag{2.3}$$

The vertical corrector CV has a *skew* sextupolar term in the magnetic field:

$$B_x = \tfrac{1}{2} s_{CV} (x^2 - y^2)$$
$$B_y = -s_{CV} xy \tag{2.4}$$

that gives the *normal* quadrupolar gradients:

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = -s_{CV} y \tag{2.5}$$

and the *skew* quadrupolar gradients:

$$\frac{\partial B_y}{\partial y} = -\frac{\partial B_x}{\partial x} = -s_{CV} x \tag{2.6}$$

These equations show that horizontal and vertical displacements Δx and Δy of the beam trajectory inside the "C" give rise to additional quadrupolar terms to the magnetic structure of the rings *changing the betatron tunes* (normal quadrupole terms) *and coupling the betatron motions* (skew quadrupole terms).

3. Tune shift with vertical displacement at the "C" correctors

Usually we use two "C" vertical correctors and the first two neighbouring Skew correctors to make a vertical closed bump at the Interaction Points.

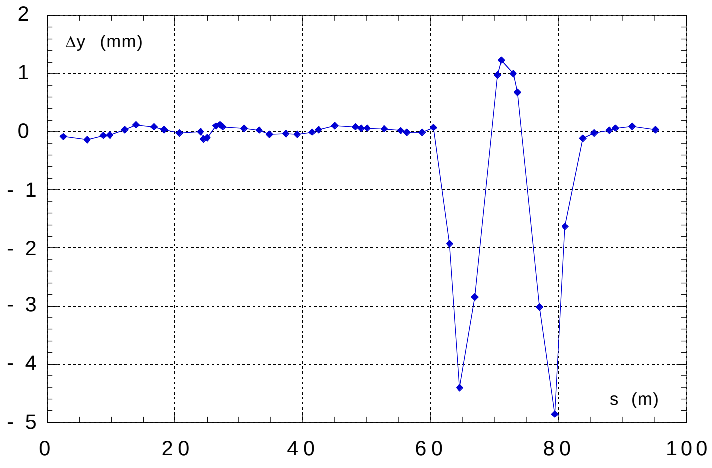

Figure 3 - Measured vertical closed bump Δy@IP2=+1 mm: the "C" positions are near the two negative peaks.

In the vertical bump around IP2 (Fig. 3) the vertical displacement at the “C” position is proportional to the vertical displacement at IP2:

$$\Delta y@"C"\cong -4.5\cdot \Delta y@IP2 \tag{3.1}$$

and from Eq. (2.5) we obtain the change of the quadrupolar integrated term provided by the corrector magnet:

$$\Delta k_{CV}\cdot L_{mag} = \frac{1}{B_0\rho} S_{CV}\cdot \Delta y \tag{3.2}$$

that changes the betatron tunes by the known relation:

$$\Delta Q_{x,y} = \mp \tfrac{1}{4\pi}(\Delta k\cdot L)\beta_{x,y} \tag{3.3}$$

The vertical displacement Δy is proportional to the current variations in the “C” and therefore, by Eq. (1.2), the sextupolar term S_{CV} is not constant when varying the bump, but it depends linearly on Δy. This explain the quadratic behavior of the tune shift with Δy found out from the measurements.

The measured [3] and calculated data from Eq. (3.3) in one case for the electron ring, listed in Table I, are compared in Figure 4.

Table I - Vertical Bump around IP2 [3]: currents in the “C” vertical corrector, displacements at IP2 and at “C”, tune shift measured and calculated

step	I@cdves201 (A)	I@cdvel201 (A)	Δy@IP2 (mm)	Δy@C (mm)	ΔQx meas	ΔQy meas	ΔQx calc	ΔQy calc
-7.00	-101.70	-92.5	-2.00	11.000	-0.016700	0.010400	-0.018899	0.0080997
-6.00	-90.17	-81.3	-1.78	9.7900	-0.013100	0.0087000	-0.014856	0.0063669
-5.00	-78.64	-70.2	-1.43	7.8650	-0.0098000	0.0067000	-0.010357	0.0044387
-4.00	-67.11	-59.0	-1.14	6.2975	-0.0070000	0.0044000	-0.0070292	0.0030125
-3.00	-55.59	-47.9	-0.860	4.7300	-0.0045000	0.0034000	-0.0043306	0.0018560
-2.00	-44.06	-36.7	-0.570	3.1350	-0.0025000	0.0024000	-0.0022413	0.00096054
-1.00	-32.53	-25.6	-0.285	1.5675	-0.0011000	0.00020001	-0.00080612	0.00034548
0.00	-21.00	-14.4	0.00	0.0000	2.0266e-09	0.00040000	0.0000	0.0000
1.00	-9.47	-3.30	0.300	-1.6500	0.00039999	-0.00040000	0.00018643	-7.9901e-05
2.00	2.06	7.85	0.595	-3.2725	0.00039999	-0.00040000	-0.00028683	0.00012293
3.00	13.59	19.0	0.890	-4.8950	-0.00020000	-0.00060001	-0.0014112	0.00060479
4.00	25.11	30.1	1.17	-6.4625	-0.0013000	-0.00020000	-0.0031597	0.0013542
5.00	36.64	41.3	1.48	-8.1400	-0.0021000		-0.0056131	0.0024056
6.00	48.17	52.5	1.77	-9.7625	-0.0045000		-0.0086907	0.0037246
7.00	59.70	63.6	2.05	-11.302	-0.0092000		-0.012329	0.0052840

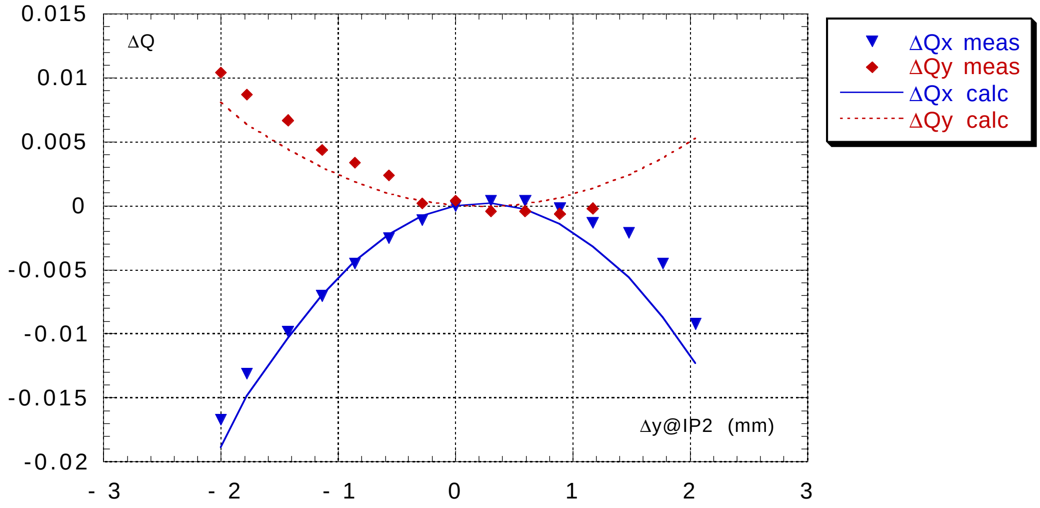

Figure 4 - Tune shift dependence by the displacement at IP2 obtained using the "C" correctors (electron ring working point: 5.12 - 5.17, wiggler on [3]).

Considering the thin lens approximation implied in Eq. (3.3) the agreement is very good and the dependence of the tunes on the vertical bumps at the Interaction Points is quite well explained by the "C" vertical corrector skew sextupolar term.

4. Conclusions

More work has to be done to study the influence of the normal and skew sextupolar terms of the horizontal and vertical "C" corrector, both for non-linearities and for coupling, but the existence of a non-negligible sextupole of the "C" gives a satisfactory answer to the effects of coupling and tune dependence from the vertical bump around the Interaction Regions of DAFNE.

References

[1] B. Bolli et al. *Measurements on TESLA "C" correctors prototype for the DAFNE Main Rings*. DAFNE Technical Note - MM-17, 1996.

[2] G. Benedetti. *Il modello dell'ottica lineare di Dafne*. Tesi di Laurea, Università di Firenze, in preparation.

[3] DAFNE Team. *DAFNE Logbook*.- February 24, 2001.

[4] S. Guiducci, M. Preger. *Calibration constants and nominal set points for the Day-one lattice of the DAFNE Main Rings*. DAFNE Technical Note - C-18, 1997.