DAΦNE TECHNICAL NOTE
INFN - LNF, Accelerator Division

Frascati, Feb. 4, 1991
Note: **G-4**

THE KICKER COUPLING IMPEDANCES

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1. INTRODUCTION

Two important parameters for the beam stability are the longitudinal and the transverse impedances, Z_L and Z_T. The impedance concept of the machine comes from the schematization of the interaction between a perturbation in the beam and the surrounding medium. Such an interaction produces fields that can provide an increase or a damping of the perturbation itself, having so instability or stability, respectively. Thus, the stability depends both on the amplitude and on the phase of these fields with respect to the perturbation amplitude. In summary, the accelerator can be assimilated with to a feedback circuit whose reaction is represented by the impedance of the machine.

This way we can define the longitudinal impedance in terms of the ratio between the averaged longitudinal field and the perturbed current which produces this field:

$$Z_L = \frac{\oint E_L ds}{I} \qquad [\Omega] \tag{1}$$

while the transverse impedance is the ratio between the averaged transverse electromagnetic force per unity charge and the perturbed current dipole momentum $I\delta$

$$Z_T = \frac{j}{\beta I \delta} \oint (E + V \times B)_T \, ds \qquad [\Omega/m] \tag{2}$$

$\beta = v/c$, **E** and **B** are the electric and the magnetic field, respectively, and, of course, cause and effect are related to the same point.

By means of a generalized definition of these quantities, we can use some differential relationships between them. Such a definition requires to keep distinct the points x_o and x_1, where the cause (current) and the effect (averaged field), respectively, take place. Thus, we define the transimpedances $Z_L(x_o,x_1)$ e $Z_T(x_o,x_1)$ as:

$$Z_L\,(x_o,x_1) = \frac{\oint E_L(x_o,x_1)\,ds}{I(x_o)} \qquad [\Omega] \qquad (3)$$

$$Z_T\,(x_o,x_1) = \frac{\oint\ [E(x_o,x_1)+V\times B(x_o,x_1)]_T\ ds}{\beta I(x_o)\,\delta} \qquad [\Omega/m] \qquad (4)$$

This way the usual impedances can be obtained from (3) and (4) for $x_o = x_1$. It is well known from the literature that the relationship between Z_L e Z_T is the following:

$$Z_T = \frac{c}{\omega}\,\frac{\delta^2\ Z_L}{\delta x_o\ \delta x_1} \qquad [\Omega/m] \qquad (5)$$

2. THE IMPEDANCES Z_L AND Z_T

Let us consider as a first example a kicker of the symmetric kind, that is a kicker for which its right side is specular of the left one. Let us define a coordinate system x orthogonal to the plane of symmetry. It is easy to see that a perturbed current located at the center of the kicker produces a magnetic field whose flux does not link with the kicker circuit. Different it is the situation if the perturbation is offset of a position x_o . In this case we can clearly see that, being lost the symmetry, there will be a flux linking with the kicker circuit, and that an interaction between the latter and the perturbation will occur. In conclusion, the longitudinal impedance will not be zero.

The following *electrotechnical* procedure, which is very simple as well, gives us some immediate results. Let us consider an offset current in x_o and let us put in the center an equal current but opposite in sign (we have seen that this one does not contribute to the impedance); these two currents represent a loop that couples with the kicker circuit by means of the mutual inductance $M\ (x_o)$. The flux linked with the kicker will be:

$$\phi(x_o) = M(x_o)\quad I(x_o) \qquad [Wb] \qquad (6)$$

In the kicker circuit, such flux generates an induced voltage. Consequently, an induced current I_k which is given by:

$$I_k = \frac{j\omega \, M(x_o)}{Z_k} I(x_o) \qquad [A] \qquad (7)$$

where Z_k is the kicker impedance (the one seen by the loop).

In turn, such current generates a flux which links with the loop creating a voltage V given by:

$$V(x_o, x_1) = \frac{\omega^2 \, M(x_o) \, M(x_1)}{Z_k} I(x_o) \qquad [V] \qquad (8)$$

Therefore, the longitudinal transimpedance is:

$$Z_L(x_o, \, x_1) = \frac{\omega^2}{Z_k} M(x_o) \, M(x_1) \qquad [W] \qquad (9)$$

while, according to (5), the transverse transimpedance is

$$Z_T(x_o, \, x_1) = \frac{c\omega}{Z_k} M'(x_o) \, M'(x_1) \qquad [\Omega/m] \qquad (10)$$

where the prime denotes the derivative with respect to x.

From the above quantities we get the usual longitudinal and transverse impedances.

A more accurate analysis, which allows for the finite length "l" of the kicker, leads to a *transit time factor* :

$$G = \left(\frac{\sin(\theta/2)}{\theta/2} \right)^2 \qquad (11)$$

where θ is the signal phase change during the transit time in the kicker, and it is defined by the following relation

$$\theta = \frac{\omega l}{\omega_o R} \qquad (12)$$

where ω/ω_o is equal to the harmonic number n and n-Q, for the longitudinal and transverse mode, respectively. We note that the square appearing in (11) is due to the twofold action of the beam on the kicker circuit and of the kicker circuit back on the beam.

In some cases the kicker is placed outside the vacuum chamber, which is formed by a ceramic wall coated by a thin metallic layer of thickness s. A crude approximation suggests that the coupling impedance must be affected by a screening factor F given by the following equation:

$$F = \exp\left(\frac{-2s}{\sigma}\right)$$

where σ is the skin depth of the metal.

3. GENERAL CONSIDERATION

The angle α charactering the kicker strength is given by the expression

$$\alpha = \frac{eV}{\beta E}\,\frac{cM}{a\zeta_k} \tag{14}$$

where V is the voltage delivered by the power supply on the impedance ζ_K, M^{-1} is the reluctance of the magnetic circuit, E is the total particle energy, and "a" is the horizontal transverse dimension of the kicker loop. Note that:

$$M \approx M'(x)\ a \tag{15}$$

so that we may write

$$\alpha \approx \frac{eV}{\beta E}\,\frac{cM'}{\zeta_k} \tag{16}$$

According to (9), (10) and (11), the longitudinal and transverse coupling impedances are written as

$$\frac{Z_L}{n} = 4\ [cM(x)]^2 \frac{\beta^2}{l^2 n Z_k}\ \sin^2 \frac{nl}{2R} \qquad [\Omega] \tag{17}$$

$$Z_T = 4\ [cM'(x)]^2 \frac{\beta R}{l^2 (n-Q) Z_k}\ \sin^2 \frac{(n-Q)l}{2R} \qquad [\Omega/m] \tag{18}$$

We may write that

$$M(x) = M'(0)(d+x) \tag{19}$$

where the dimension "d" accounts for the asymmetry of the kicker, if any. Furthermore $M'(x) \approx M'(0)$.

Allowing for the above considerations and by combining (17) and (18) with (14) and (15) we may write

$$\frac{Z_L}{n}\ , \quad Z_T \propto \frac{\zeta_k^2}{Z_k l^2}\ , \tag{20}$$

where ζ_k is the impedance seen by the thyratron at its working frequency (say the fundamental harmonic), while Z_k is the output impedance (seen toward the thyratron) at the perturbation frequency.

It is easy to see that it is necessary to slow down ζ_k and enhance Z_k and l, in order to optimize the characteristics as well as the stability margin of the kicker.

4. WINDOW-FRAME AND C-TYPE KICKERS

The mutual inductance of two monoloop electric circuits is the inverse of the reluctance of the magnetic circuit linking each other. Allowing for a window-frame kicker (see Fig.1) we may give a simple expression of the mutual inductance. Let us first note that the two electric circuit are formed by the beam current placed in x and the opposite current placed in 0. The reluctance we are looking for is one resulting from the series of the gap reluctance and the jokes reluctance. Neglecting the latter $(m \gg m_o)$ we have in linear approximation

$$M(x) = \mu_o \frac{l\,x}{h} = \frac{Z_o l\,x}{hc} \qquad [H] \tag{21}$$

where h is the eight of the gap and Z_o is the free space transimpedance. Furthermore, the following relation holds:

$$M'(x) = \frac{Z_o l}{ch} \qquad [H/m] \tag{22}$$

As a conclusion, allowing for the results of the previous section and for (21) and (22) we get:

$$\frac{Z_L(x)}{n} = 4\frac{Z_o^2}{Z_k}\left(\frac{x}{h}\right)^2 \frac{\sin^2(nl/2R)}{n} \qquad (23)$$

$$Z_T(x) = 4R\frac{Z_o^2}{Z_k}\left(\frac{1}{h}\right)^2 \frac{\sin^2[(n-Q)l/2R]}{n-Q} \qquad [\Omega/m] \qquad (24)$$

where R is the machine mean radius. Note that for a window-frame kicker the longitudinal impedance is zero for a beam placed in the center $(x=0)$.

In the case of a "C" type magnet, the formula for the transverse impedance is the same. For the longitudinal impedance, we must replace the quantity x by the quantity $x-\Delta$, where Δ is the distance from the centerof the so called neutral point. This is the point such that a current placed in it creates a flux which does not limit the kicker circuit.

In a more general case of a kicker without magnetic circuit, the considerations done in section 2. and 3. still hold. However, we are not able to give explicit formulas for the impedances, since the mutual inductance $M(x)$ and/or the reluctance are not easy to compute. An alternative method is the experimental determination.

5. REMARKS AND ESTIMATES

Now let us consider the period during the inhibition time of the kicker. The inactivity condition of kicker leads that the impedance seen by the beam, that appears in (14) and (15) results, with a good approximation, given by:

$$Z_k = -j\ Z_k\ \mathrm{ctg}(kL \qquad (16)$$

where L is the length of the cable and k is the propagation contact in it. This dependence leads the impedance to vary periodically between very large and very small values. So that, according to the schematic behaviour of $|Z_L/n|$ and $|Z_T|$ shown in Fig. 3, these small values would be very dangerous for the stability. The distance between the maxima increases as the length of the cable decreases.

It is worth emphasizing that even in the case of a very short cable, the thyratron capacitance is more and more important. This capacitance, coupled with the kicker inductance, may indeed resonate producing a phenomenon similar to the one of a mismatched cable.

As an example we give the numerical value of the longitudinal coupling impedance at a frequency of 2.4 GHz (n=800) for a typical C-kicker placed outside a coated ceramic wall. Assuming an aspect ratio $\Delta/h = 0.5$, an impedance $Z_k = 12.5\ \Omega$, and a screening factor $F_{dB} = -20$ dB, we get

$$\frac{Z_L}{n} = 0.12\ \Omega.$$

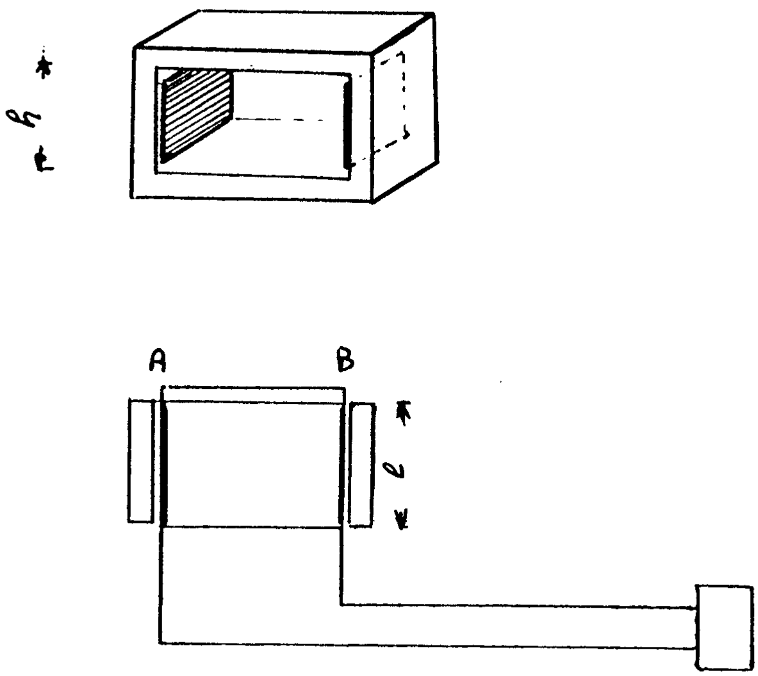

Fig. 1

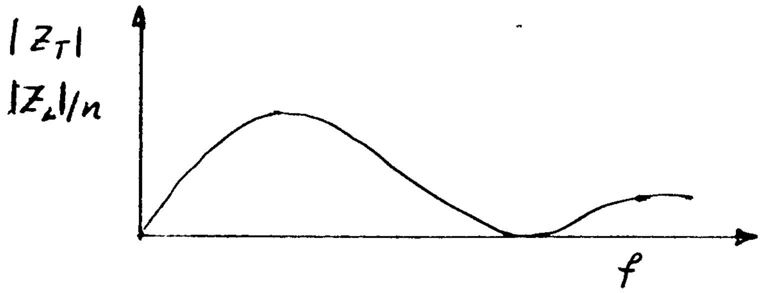

Fig. 2

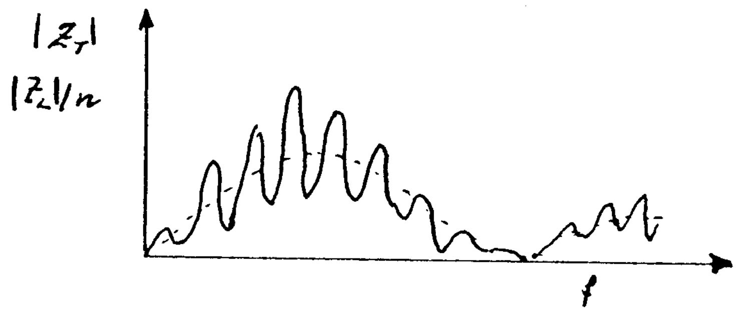

Fig. 3