DAΦNE TECHNICAL NOTE

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Note: **G-9**

ENERGY LOSS IN TAPERED TRANSITIONS IN THE DAΦNE VACUUM CHAMBER

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Introduction

The energy loss and the energy spread of a bunch can be produced by variations in cross-sectional shape of the DAΦNE vacuum chamber as well as by resonant structures such as RF cavities, bellows, vacuum ports, etc. In order to reduce the loss factor of a bunch passing through a cavity or any beam pipe discontinuity we can use a "taper" - a gradual transition between two cross sections of the beam pipe from a smaller radius 'a' to a larger radius 'b' (Fig. 1, Fig. 2).

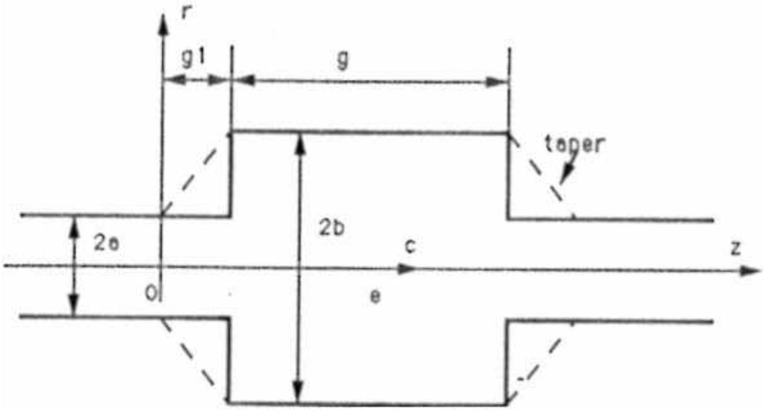

Fig. 1 - Pill-box cavity.

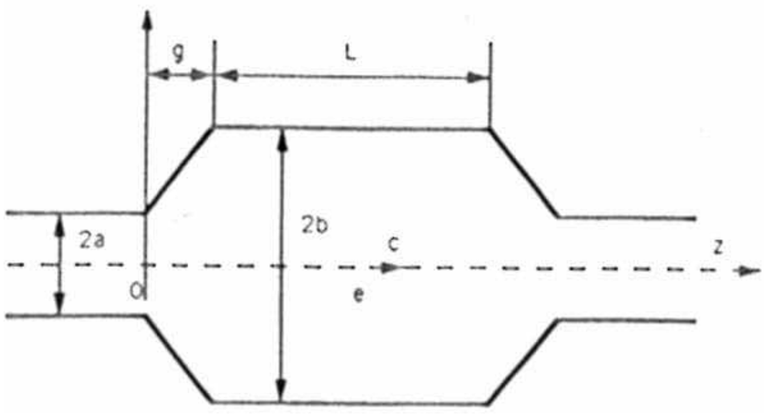

Fig. 2 - Tapered cross-sectional variation of a vacuum chamber.

An infinitely long symmetric taper reduces the losses to zero. But, since there are practical limitations, we are not allowed to have a very long taper. So there is a task to choose a profile of the taper to reduce the loss factor as much as possible, for given length g1 of this taper.

In this paper we will show a possible way for analytical calculations of a cavity loss factor, with a taper of an arbitrary form. We will also consider applications of tapers to reduce energy loss and energy spread of a bunch due to cross-sectional variations in the DAΦNE vacuum chamber.

1. **Tapered cross section variations in DAΦNE vacuum chamber**

The transition between beam tubes with different radii (Fig. 2) was considered for the DAΦNE project parameters: rms bunch length $\sigma = 3$ cm, b = 2 cm, a = 1 cm (and a = 1.5 cm), number of particles in a bunch N = $9*10^{10}$.

For the DAΦNE case, the length L is rather long (L >100 cm). Numerical simulations show that the energy loss factor does not depend on L if L is longer than 30 cm. In our simulations L = 30 cm in order to reduce the CPU time. The length of a taper `g` was varied.

According to the definition, the loss factor k_l for a particular case of a gaussian longitudinal distribution of the bunch density, with rms length σ is [1]:

$$k_l = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} Z_l(\omega,r)\, e^{-k^2\sigma^2} \qquad (1)$$

with $k = \omega/c$, and $Z_l(\omega,r)$ - the longitudinal impedance of the element of a vacuum chamber.

Below the cut-off frequency $Z_l(\omega,r)=0$. Above the cut-off frequency kb>2.405 and for DAΦNE parameters in the integrand of (1)

$$e^{-k^2\sigma^2} < e^{-2.405^2*3^2}.$$

So the loss factor should be very small. It was confirmed by numerical simulations with the code TBCI. The results are:

$k_l = -5*10^{-6}$V/pC for a = 1 cm and $k_l = -1.9*10^{-5}$V/pC for a = 1.5 cm.

Practically these values are close to zero. But, as we can see from the Fig. 3, due to wake-fields, the first part of a bunch is decelerated and the other part is accelerated. It means that the bunch energy spread increases even if the average energy loss is equal to zero. We estimated the rms energy spread for a monochromatic bunch as:

$$\sigma_E = \sqrt{\frac{\int_{-\infty}^{+\infty} \left[\Delta E(\tau)\right]^2 f(\tau)\, d\tau}{\int_{-\infty}^{+\infty} f(\tau)\, d\tau}} \qquad (2)$$

with energy loss of a particle $\Delta E(t)$ and the distribution function of particles in the bunch $f(\tau)$ taken from results of the code TBCI. For non-tapered cavity ($g = 0$) $\sigma_E = 0.48$ keV for $a = 1$cm and $\sigma_E = 0.19$ keV for $a = 1.5$ cm.

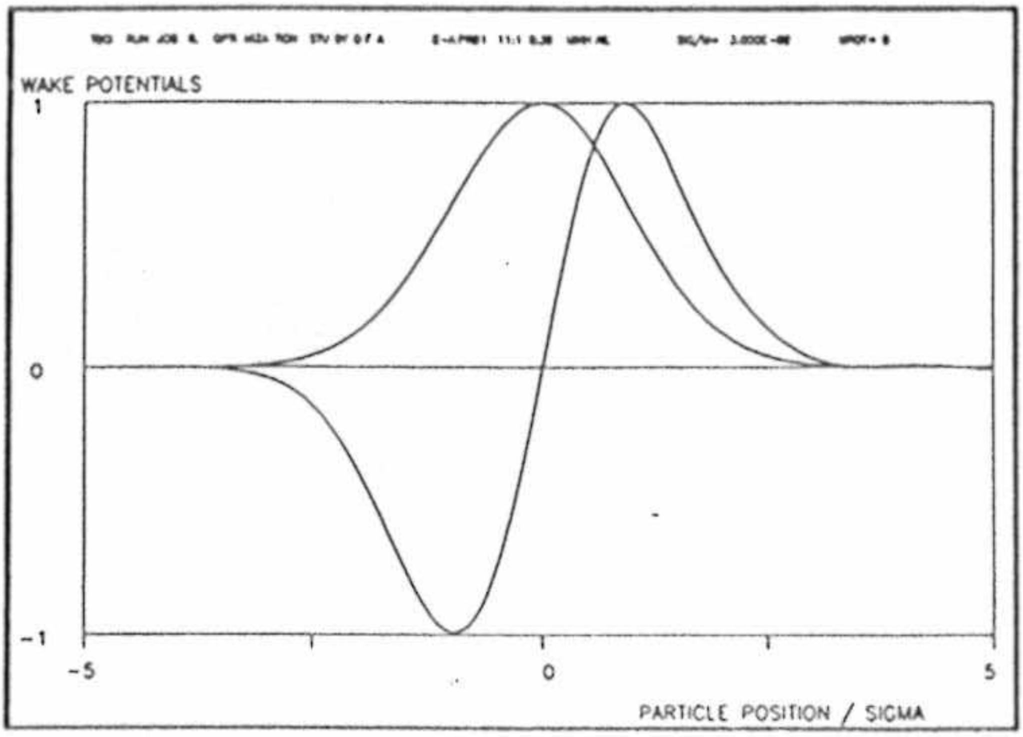

Fig. 3 - Results of the TBCI code: Bunch current distribution and associated longitudinal wake.

The use of a taper decreases the amount of energy which is radiated by a single particle. So we can expect that in the case of a taper, the energy spread of a bunch will be smaller than it would be while passing through a nontapered cavity. Numerical results for different `g` are presented in Fig. 4. We can see that a taper of reasonable length (for example, g = 5 cm) decreases the energy spread by a factor 3.

During the injection, the bunch length σ could be less than 3 cm. It would be very important to know how the changes of σ influence the bunch energy loss. In Fig. 5 the dependences of the loss factors vs. the bunch length, for different taper length are shown. The important feature is that the sensitivity of the bunch energy loss to the changes of the bunch length falls down as the length of the taper grows.

It is worth noting that the energy spread is much more higher for shorter bunches. For σ = 1 cm in given structures σ_E is 10 times more than for σ = 3 cm. But still the energy spread is less for the tapered cavity, and the dependence has the same form as shown in Fig. 4.

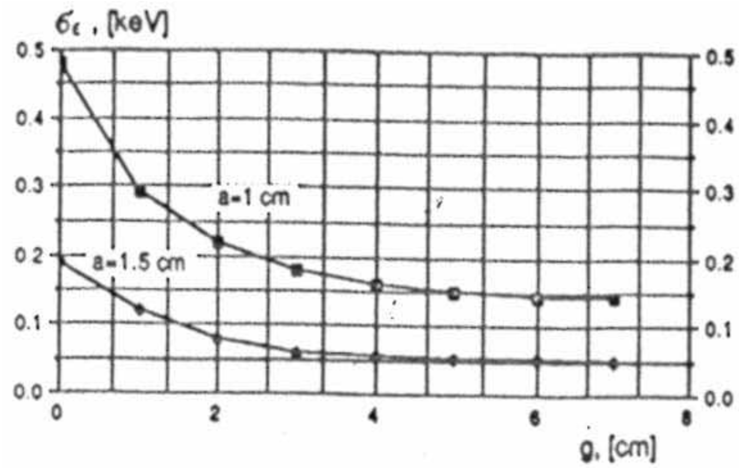

Fig. 4 - Dependence of rms energy spread on the taper length.

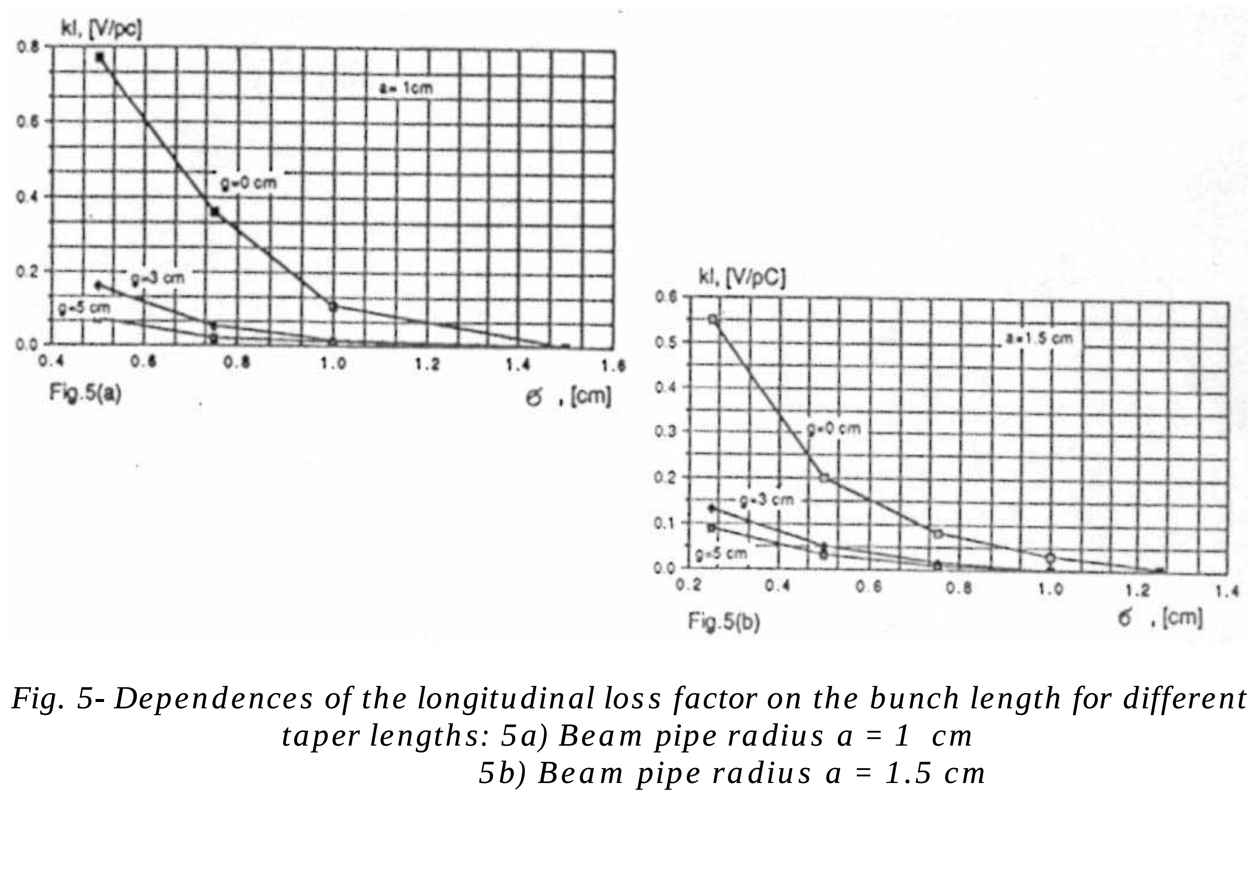

Fig. 5- Dependences of the longitudinal loss factor on the bunch length for different taper lengths: 5a) Beam pipe radius a = 1 cm
5b) Beam pipe radius a = 1.5 cm

We can conclude that:

1. The average energy loss of a bunch due to variations in cross-sectional shape (as shown in Fig. 2) for DAΦNE parameters is close to zero.

2. A taper with a reasonable length (3÷5 cm) allows to reduce the energy spread of a bunch approximately by a factor 3.

3. The use of the taper helps to keep the loss factor close to zero when the bunch length is reduced (up to σ=0.7 cm).

4. The vacuum chamber with a=1.5 cm has the energy spread of a bunch by a factor 3 smaller than the vacuum chamber with a = 1 cm.

2. **The loss factor for a tapered cavity**

In [2] a low loss cavity for the DAΦNE main ring was proposed. The gradual linear taper was included in the geometry. This taper allows to reduce the energy loss drastically. It is necessary to investigate whether it is possible to get any additional decreasing in the loss factor by choosing a profile form of the taper. Also it is worth noting that the problem of analytical calculation of the loss factor for an arbitrary form taper has its own meaning for acceleration techniques.

As the first step, let us consider the loss factor for a crosstalk between cavities (Fig. 6). The longitudinal impedance of an element of a vacuum chamber $Z_l(\omega, r)$:

$$Z_l(\omega,r) = \frac{1}{e} \int_{-\infty}^{+\infty} dz\, e^{-ik/\beta} E_{\omega,r}(r,z) \qquad (3)$$

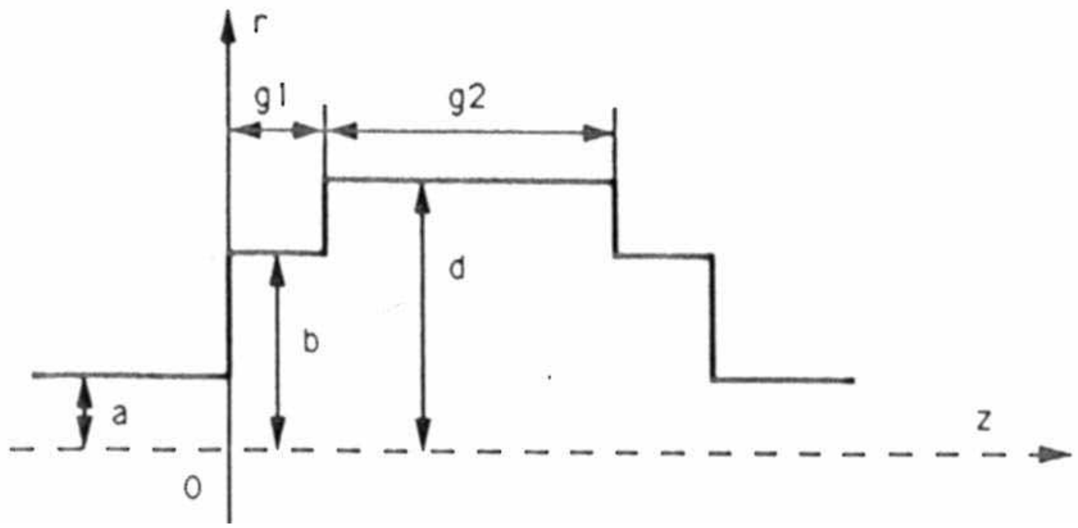

Fig. 6 - Crosstalk between cavities.

It was shown using the diffractive model [1] that the impedance of a crosstalk is:

$$Z_l = Z_i(a,b,g) + \delta Z_l \tag{4}$$

where $Z_i(a, b, g)$ is the impedance of the cavity with the radius `b` and the length g = 2g1 + g2 with the beam pipe of the radius `a`. The second term δZ_l gives a correction due to the second cavity.

For the single cavity (Fig. 1):

$$E_{\omega,r}(\omega, r) = \frac{ie}{2c}\int_{-\infty}^{+\infty} dq e^{iqz} J_0(rs)[H_0^{(1)}(bs)-H_0^{(1)}(as)](1-e^{ig(k-q)}) \tag{5}$$

We are considering the azimuthally symmetric structures with a particle travelling along its z axis with the velocity of light. In this case it is sufficient to consider the monopole mode. For this mode the result of integration of (3) does not depend on r. Choosing r = a for the integration:

$$Z_l(k) = \frac{2}{c}\int_{-\infty}^{+\infty} \frac{dq}{k-q} J_0(as)[H_0^{(1)}(as)-H_0^{(1)}(bs)]\sin^2(\frac{g}{2}(k-q)) \tag{6}$$

Since of k_l is a real value, we are interested in $ReZ_l(k)$:

$$R_eZ_l = \frac{2}{c}\int_{-k}^{+k} \frac{dq}{k-q} J_0(as)[J_0(as)-J_0(bs)]\sin^2(\frac{g}{2}(k-q)) \tag{7}$$

The real part of $Z_i(a,b,g)$ is given by (7). δZ_l is given by the Eq. (3) over the waves $E_{\omega,z}$ (z, a) generated at z = g1 and z = g2 + g1. For the case when there is no screening of the surface z = g1 by the surface at z = g1 + g2 :

$$E_{\omega,z}(z,a) = \frac{ie}{2c}\int_b^d dr' \int_{+\infty}^{-\infty} dq e^{iqz} J_0(as)\frac{\partial H_0^{(1)}(r's)}{\partial r'}(e^{i(k-q)g1}-e^{i(k-q)(g1+g2)}) =$$

$$= \frac{ie}{2c}e^{kg1}\int_{-\infty}^{+\infty} dq e^{iq(z-g1)} J_0(as)[H_0^{(1)}(ds)-H_0^{(1)}(bs)](1-e^{i(k-q)g2}) \tag{8}$$

Hence:

$$\delta Z_l = \frac{i}{2c}\int_{-\infty}^{+\infty} dz e^{-ik(z-g1)} \int_{-\infty}^{+\infty} dq e^{iq(z-g1)} J_0(as)[H_0^{(1)}(ds)-H_0^{(1)}(bs)](1-e^{i(k-q)g2})=$$

$$= \frac{i}{2c}\int_{-\infty}^{+\infty} dx e^{-ikx} \int_{-\infty}^{+\infty} dq e^{iqx} J_0(as)[H_0^{(1)}(ds)-H_0^{(1)}(bs)](1-e^{i(k-q)g2}) \qquad (9)$$

The real part of δZ_l :

$$R_e\delta Z_l = \frac{2}{c}\int_{-k}^{+k} \frac{dq}{k-q} J_0(as)[J_0(bs)-J_0(ds)]\sin^2\left(\frac{g2}{2}(k-q)\right)=$$

$$= \frac{2}{c}\int_{-k}^{+k} \frac{dq}{k-q} J_0(as)[J_0(as)-J_0(ds)]\sin^2\left(\frac{g2}{2}(k-q)\right) -$$

$$- \frac{2}{c}\int_{-k}^{+k} \frac{dq}{k-q} J_0(as)[J_0(as)-J_0(bs)]\sin^2\left(\frac{g2}{2}(k-q)\right) \qquad (10)$$

If we compare Eq. (10) and Eq. (7) we will see that:

$$R_e\delta Z_l(k) = R_e\delta Z_{l_1}(k) - R_e\delta Z_{l_2}(k) \qquad (11)$$

where $R_e\delta Z_{l1}(k)$ is the real part of the impedance of the cavity with the radius \`d\`, length g2 and beam pipe radius \`a\`. $R_e\delta Z_{l2}$ (k) is the real part of the impedance of the cavity with radius \`b\`, length g2 and beam pipe radius \`a\`.

So, the loss factor of a crosstalk k_l^{ct} must be equal to :

$$k_l^{ct} = k_l^1 + k_l^2 - k_l^3 \qquad (12)$$

where k_l^i corresponds to i-cavity presented in Fig. 7.

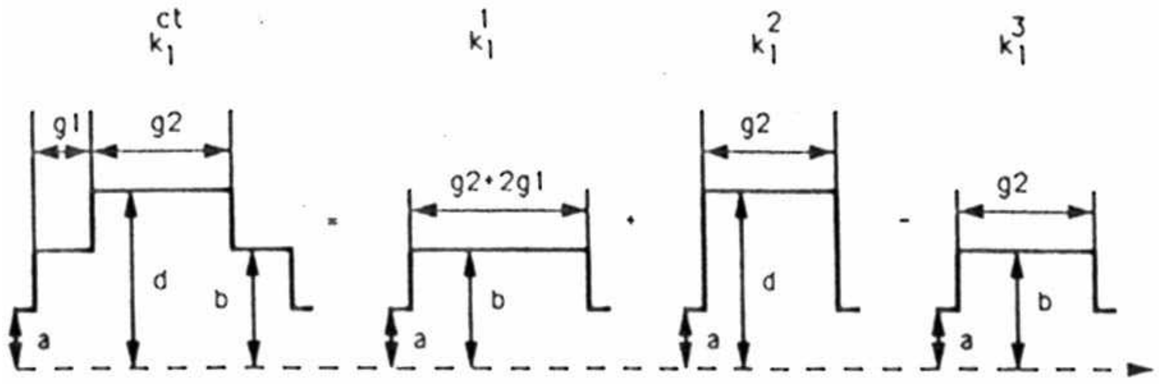

Fig. 7 - Loss factor of a crosstalk

The loss factor can be calculated by using the formulae of the diffractional model. For the cavity presented in Fig. 1 (without taper):

$$k_l = \frac{1.154}{a}\sqrt{\frac{g}{\pi\sigma}}\,, \qquad \text{if } P<1 \qquad (13)$$

$$k_l = \frac{2}{\sigma\sqrt{\pi}}\ln\left[1+\frac{b-a}{a}\sqrt{\frac{2g\sigma}{2gs+(b-a)^2}}\,\right], \quad \text{if } P>1 \qquad (14)$$

$$P = \frac{2g\sigma}{(b-a)^2} \qquad (15)$$

We made numerical calculation for some examples of crosstalks using the TBCI code. There is a good agreement between (12)-(15) and the results of simulations. Table I gives loss factors for crosstalks calculated by TBCI (k^{TBCI}) and using eqs.(12)÷(15). All parameters in Table I correspond to Fig. 6, δ is the relative error of analytical calculations in comparison with TBCI results.

TABLE I - Loss factors for crosstalks

σ, cm	a, cm	b, cm	d, cm	g1, cm	g2, cm	k^{TBCI} V/pC	k_l, V/pC	[δ],%
0.3	1.5	2.5	4	1	8	2.141	1.981	8
0.3	1	2	3	1	4	2.380	2.163	9
1	3	5	7	5	20	0.684	0.688	<1
3	3	14.5	17	5	88	0.427	0.427	0

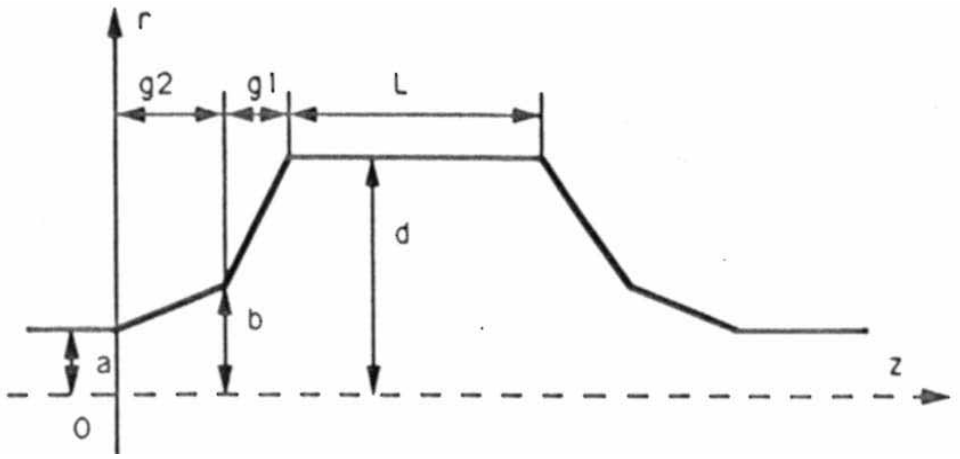

Fig. 8 - Tapered cavity (taper profile consists of two straight lines).

Let us consider the taper that consists of two straight lines (Fig. 8). For this tapered cavity the diffractional model gives:

$$E_{\omega,r}(z,r) = \frac{ie}{2c}\int_a^b dr' \int_{-\infty}^{+\infty} dqJ_0(as)e^{iqz}\frac{\partial H_0^{(1)}(r's)}{\partial r'}\left(e^{i(k-q)(r'-a)\cot\alpha_1} - e^{i(k-q)(L+2(g1+g2)-(r'-a)\cot\alpha_1)}\right) +$$

$$+ \frac{ie}{2c}\int_b^d dr' \int_{-\infty}^{+\infty} dqJ_0(as)e^{iqz}\frac{\partial H_0^{(1)}(r's)}{\partial r'}\left(e^{i(k-q)(g2+(r'-b)\cot\alpha_2)} - e^{i(k-q)(L+2g1+g2-(r'-b)\cot\alpha_2)}\right) \quad (16)$$

where $\cot\alpha_1 = g2/(b-a)$ and $\cot\alpha_2 = g1/(d-b)$

Calculating the impedance of the tapered cavity, according to (3) with the offset r = a we obtain:

$$Z_l^T(k) = Z_i^T(a,b,L+2g1) + \delta Z_l^T \quad (17)$$

The first term in (17) is the impedance of the cavity with radius `b`, length L + 2g1, with two tapers of length g2 and beam pipe radius `a`.The second term gives a correction due to the upper part (z>b) of the whole tapered cavity. We can show that this upper part gives the contribution to the loss factor that can be approximated by $k_{l2}^T - k_{l3}^T$, where k_{l2}^T is the loss factor in the cavity with the radius 'd', the length L, with two tapers of the length g1 and the beam pipe radius 'a'. k_{l3}^T is the loss factor in the cavity, with the radius 'b' the length L with two tapers of the length g1 and the beam pipe radius 'a'. (see Appendix).

So, the loss factor k_l^T of the tapered cavity shown in Fig. 8 is:

$$k_l^T = k_{l_1}^T + k_{l_2}^T - k_{l_3}^T \quad (18)$$

where k_{li}^T corresponds to i - tapered cavity presented in Fig. 9 .

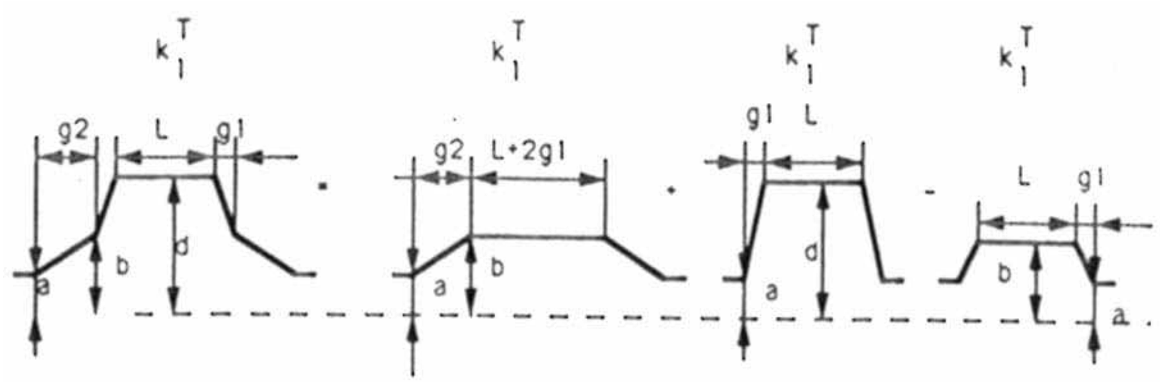

Fig. 9 - Loss factor of a taper consisting of two straight lines

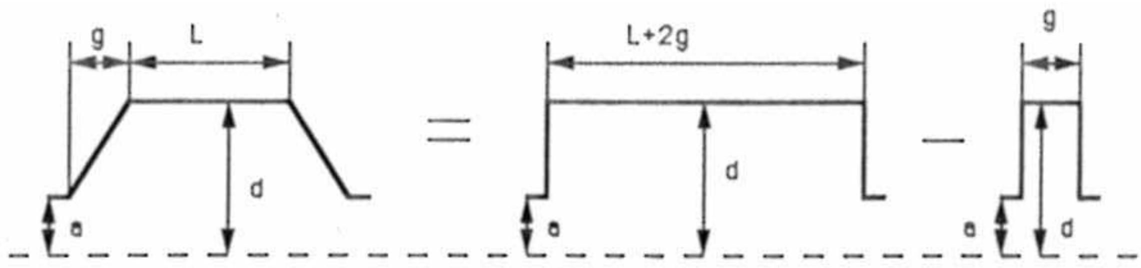

Fig. 10 - Loss factor of a linear taper

Each k_{li}^T can be calculated as the difference of loss factors of non-tapered cavity, and the cavity with the length that is equal to the length of corresponding taper (Fig.10). For this purpose the equations (12)-(15) are used.

Many numerical simulations were made using the TBCI code. We find a good agreement between the results of TBCI and the equations (12)÷(15). Table II demonstrates these results (k^{TBCI} corresponds to loss factor values calculated by TBCI; k_l - analytical values; all parameters are the same as presented in Fig. 8).

TABLE II - Loss factors for tapers consisting of two straight lines

σ, cm	a, cm	b, cm	d, cm	g1, cm	g2, cm	L, cm	k^{TBCI} V/pC	k_l, V/pC	[δ],%
1	3	7.96	17	3.7	12.3	88	1.242	1.200	3.2
0.3	1	1.5	3	1	2	40	2.493	2.567	2
0.3	1	1.5	4	1	1	40	3.656	3.631	0.6
0.1	1	1.25	1.5	0.25	0.75	20	2.449	2.124	13
0.5	2	3.8	6	2	3	40	1.482	1.518	2.4
0.5	1.5	2.5	4.5	1	2	40	1.540	1.584	3.3

But in some cases, the errors are more than 15%. It is explained by the fact that (12)-(15) have the accuracy worse than 15% in the transition region from `cavity` regime to the `step` regime in a number of cases. But the equation (18) is still valid also in these cases, if we would take k_{li}^{T} from the results of TBCI. Table III gives such examples (δ is the relative error of loss factor analytical calculation; δ^{TBCI} is the relative error of loss factor calculation when loss factors for all tapers presented in Fig. 9 are taken from the TBCI results).

TABLE III - Loss factors for tapers consisting of two straight lines

σ, cm	a, cm	b, cm	d, cm	g1, cm	g2, cm	L, cm	k^{TBCI} V/pC	δ,%	δ^{TBCI} %
0.5	1.5	3	4.5	0.5	2.5	40	1.498	20	2.5
0.5	2	2.5	3.5	0.5	2	40	0.581	25	12

In general case we can approximate the taper having an arbitrary form by some straight lines and calculate the loss factors using the simple formulae (12)-(15).

It is also possible to use methods of optimization to find the taper providing the minimal loss factor. It does not take much CPU time because (12)-(15) are very simple.

As an example, we use the method of scanning in order to find the taper consisting of two straight lines that has the minimal loss factor (for given bunch length,radius of a cavity, radius of a beam pipe and length of the taper). The loss factor of tapered cavity, presented in Fig. 11, is 18% less than one for the cavity with the straight taper having the same length. (Results were verified by TBCI code).

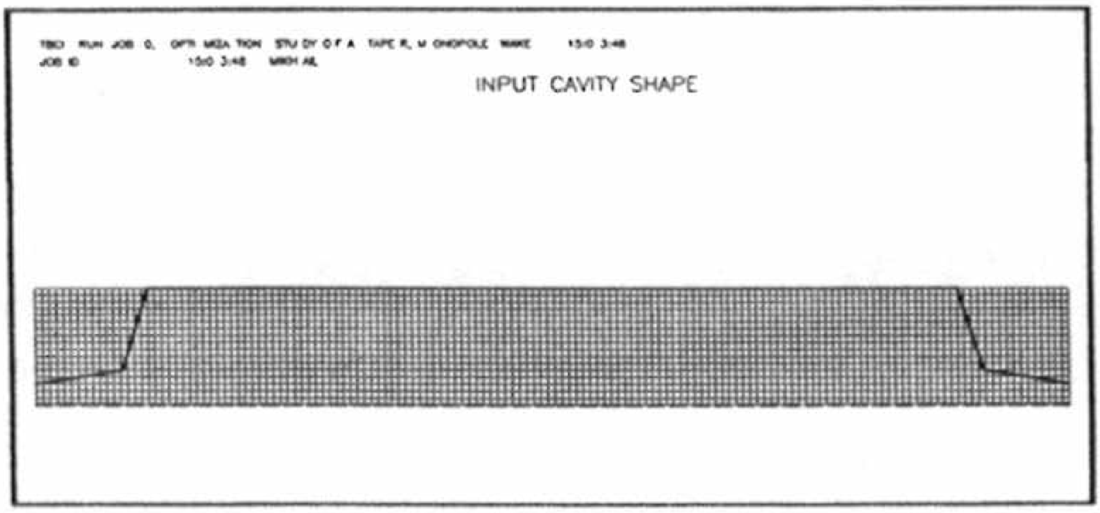

Fig. 11 - Tapered cavity having 18% smaller loss factor than for cavity with straight taper

Let us return to the case of a crosstalk (Fig. 6). For g1<<g2 and P>>1 (the regime a 'step'):

$$k_l^2 = \frac{2}{\sigma\sqrt{\pi}} \ln(\frac{d}{a}) \qquad (19)$$

$$k_l^3 = \frac{2}{\sigma\sqrt{\pi}} \ln(\frac{b}{a}) \qquad (20)$$

So:

$$k_l^2 - k_l^3 = \frac{2}{\sigma\sqrt{\pi}} \ln(\frac{d}{b}) \qquad (21)$$

is the loss factor for the cavity with the radius 'd' with the beam pipe radius 'b' for P>>1. For P>>1, the loss factor of a crosstalk is the sum of the loss factors of the cavity with radius 'b', the beam tube with radius 'a', the cavity with the radius 'd' and the beam pipe radius 'b' (Fig.12). It means that the contribution of interference of waves diffracted from the crosstalk surface to the loss factor is negligible for P>>1.

Numerical simulations show that for the proposed DAΦNE main ring cavity [2] it is valid even P>1. The loss factor of the cavity with the symmetric taper (Fig. 13) is equal to $k_l = -0.1186$ V/pC. The sum of the loss factors of the separate elements - the cavity (Fig. 14) and the taper (Fig. 15) is $k_l = -0.113$ V/pC. The contribution of interference is approximately 4%.

It would be reasonable to suppose that interference of wave diffracted from the taper, which consists of two straight lines (Fig. 8), could be also small for P>>1. And the loss factor could be equal to the sum of the loss factors of two tapers for P>>1 (Fig. 16).

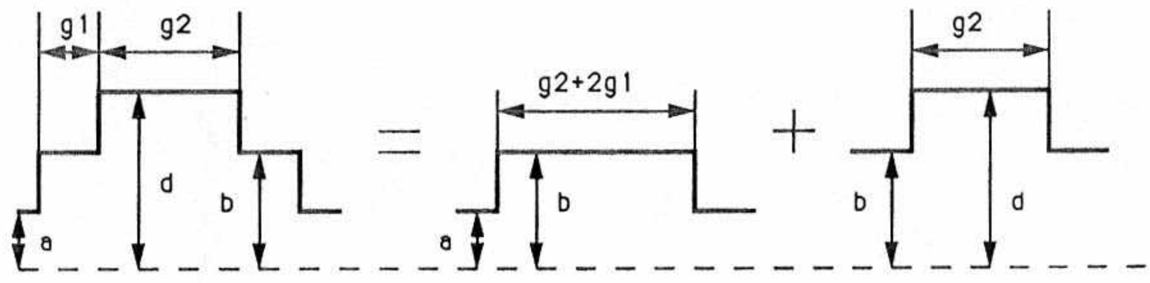

Fig. 12 - Crosstalk loss factor for the case of small interference.

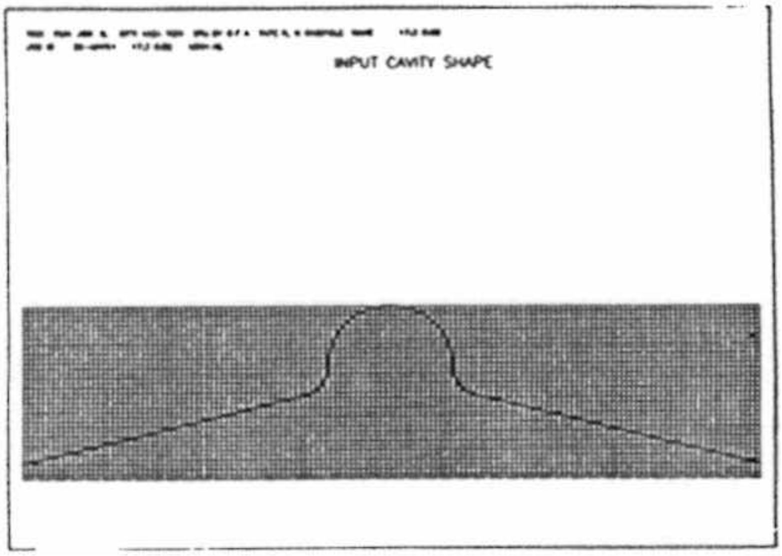

Fig. 13

RF cavity with symmetric taper.

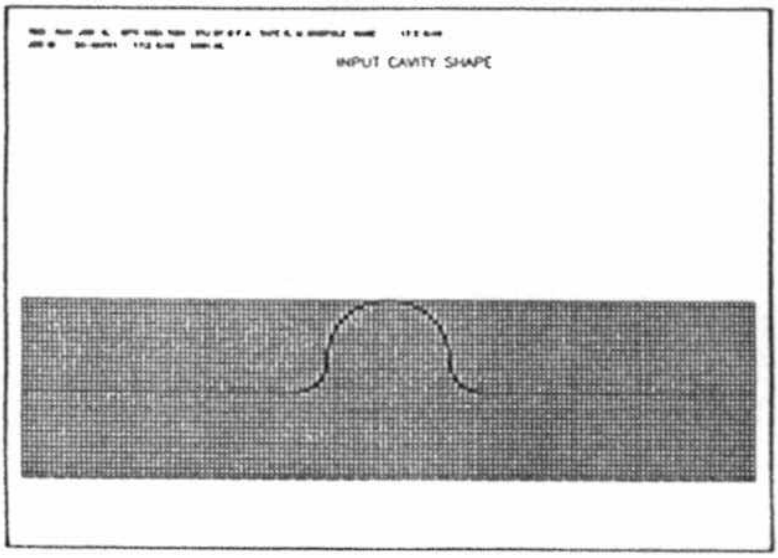

Fig. 14

RF cavity without taper.

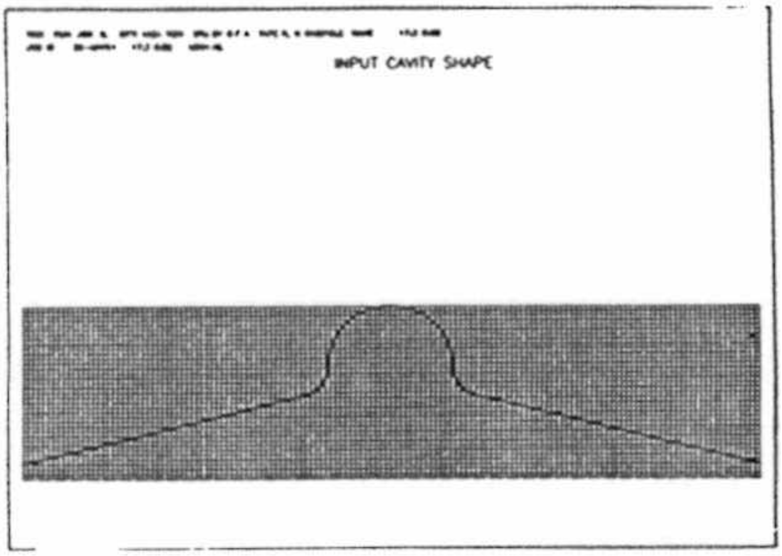

Fig. 15

Taper for RF cavity.

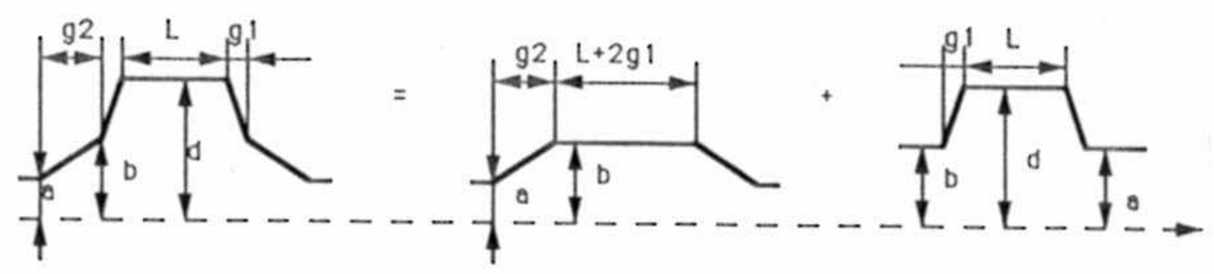

Fig. 16 - Taper loss factor for the case of small interference.

We have considered the particular case of the taper for DAΦNE cavity, with d = 17 cm, a = 3 cm, g3 + 2(g1 + g2) = 148 cm, σ = 3 cm. g1, g2 and b were varied (for all cases P>>1). For each straight taper in Fig. 16, loss factors were calculated using Eqs. (12)÷(15). The agreement is within 15% accuracy and very often much more better. Table IV presents some results.

TABLE IV – Taper loss factors for the case of small interference.

a, cm	b, cm	d, cm	g1, cm	g2, cm	L, cm	k^{TBCI} V/pC	k_1 V/pC	δ, %
3	8	17	8	22	88	0.1414	0.1411	0.2
3	11	17	4	26	88	0.110	0.123	12
3	5	17	16	14	88	0.146	0.147	1.2
3	15	17	2	28	88	0.078	0.09	12.5
3	5	17	8	22	88	0.237	0.225	5
3	7	17	25	30	38	0.082	0.08	3

So for the DAΦNE case, the loss factor of a taper consisting of two straight lines, is a simple sum of loss factors of two linear tapers. For each linear taper, the analytical formulae of the diffractional model are valid. We can divide the linear taper in two ones and find the geometry when the loss factor has a minimum. To find this solution the procedure of scanning can be used. Then we divide separately each of these new tapers and search for them the best solutions. It is possible to repeat this procedure of dividing as many times as we want.

In such a way we got some examples when a taper, consisting of a number of straight lines, had the loss factor smaller than the one for the linear taper. Fig. 17 shows the taper profile approximated by 4 straight lines. For such a taper the loss factor is less in a factor 1.8 than the loss factor of the linear taper (It is not the DAΦNE case).

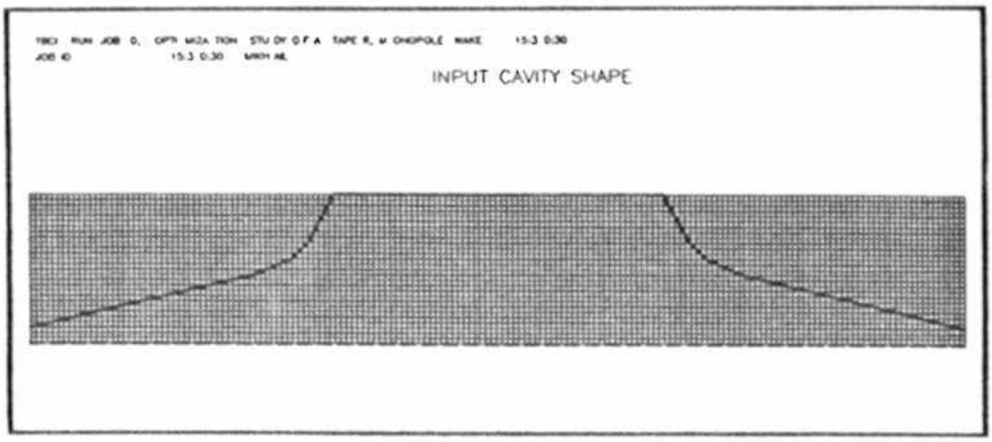

Fig. 17 - Optimized taper profile, approximated by 4 straight lines.

As far as the DAΦNE cavity tapers are concerned we can get the improvement only 10% for the short taper (g = 30 cm). For the longer one (g = 55 cm) the procedure of profile optimization does not give any additional reduction of the loss factor.

In practice the dependence of the loss factor on the linear taper length has the form presented in Fig. 18. Than we can get some reduction of the loss factor by choosing a profile only if we work somewhere at the point A. The optimization and numerical simulations show that if we work at the right part of the curve (the point B), the modification of the taper profile will not give any essential improvement of the situation. Even increasing of the taper length for the DAΦNE cavity taper by a factor 2, decreases the loss factor no more than 20%.

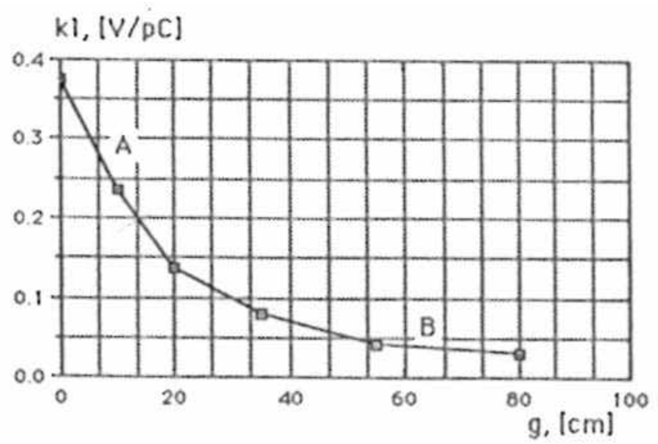

Fig. 18 - Dependence of the loss factor on the linear taper length.

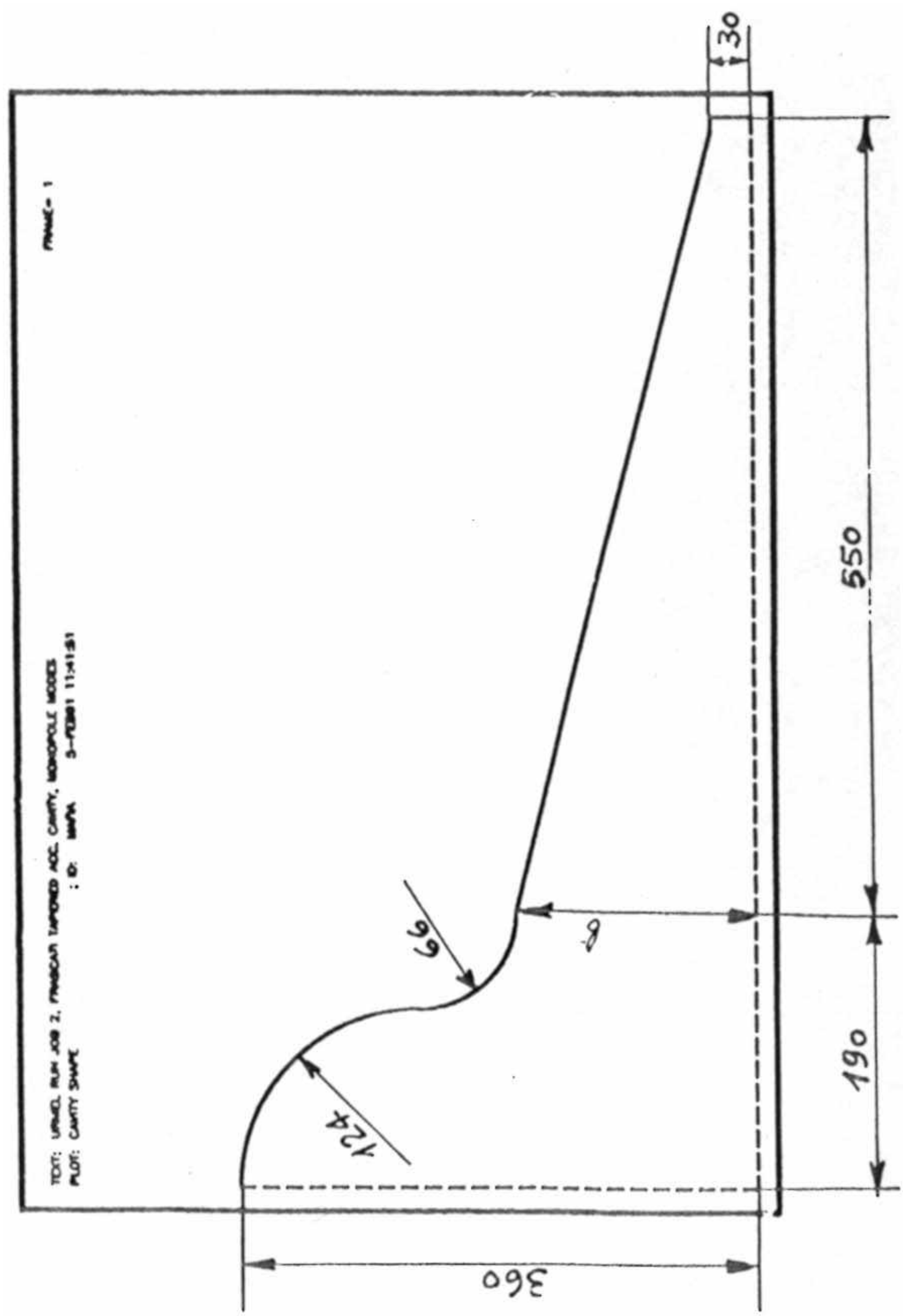

Fig. 19 - Proposed DAΦNE main ring RF cavity.

For the DAΦNE cavity (Fig. 19) we also tried to find the minimum loss factor varying the height 'b' of the taper without changing the cavity form. The loss factor was calculated separately for the taper and the cavity because the contribution of interference is practically absent. Increasing of 'b' results in decreasing of the cavity loss factor proportionally ln(1+19/b). But at the same time the contribution of the taper in the loss factor grows. And inversely decreasing of 'b' results in fast increasing of the cavity loss factor while the contribution of the taper falls down. There exists the minimum of the loss factor when b=16.5 cm. This optimal position gives only 5% improvement in comparison with b = 17 cm proposed in [2].

Another idea was to use an asymmetric taper to win in the loss factor (Fig. 20). But still the best result gives a symmetric taper. Fig. 21 demonstrates the dependence of k_l on the left taper length (The total length of two tapers is kept constant (110 cm)). According to the theorem that the loss factor does not depend on the direction of charge movement, in an arbitrary shape cavity, with equal beam pipe cross sections [3], this dependence has the symmetric form.

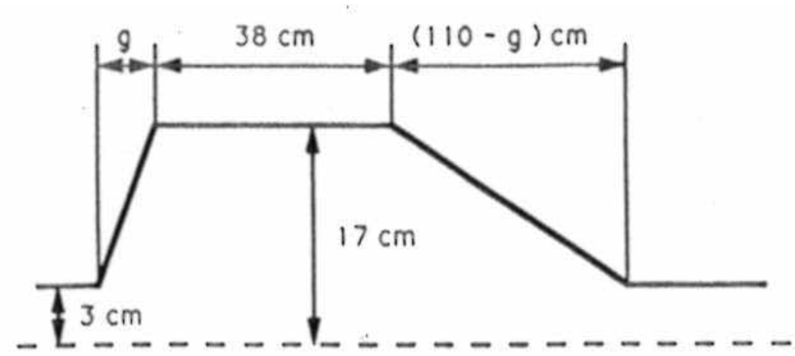

Fig. 20 - Asymmetric taper for the DAΦNE cavity.

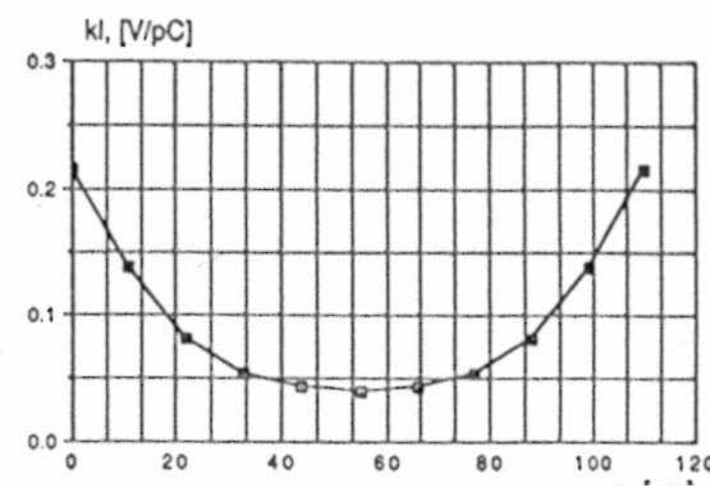

Fig. 21 - Loss factor for the asymmetric taper.

The last way to get the additional decreasing of the loss factor is to change the cavity shape. Fig. 22 shows such a shape with the taper length g=95 cm. Such a choice would allow to reduce the loss factor from -0.12 V/pC to -0.065 V/pC. But it is obvious that it would worthen the shunt impedance drastically.

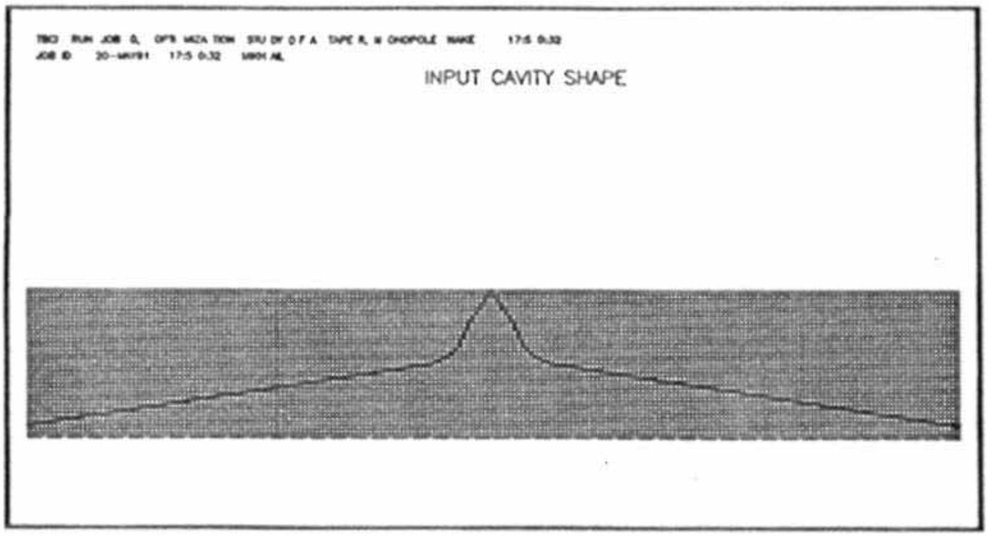

Fig. 22 - Changed cavity shape.

We can conclude:

1. An arbitrary form taper can be approximated by a number of straight lines. The loss factor of such a taper is the sum of loss factors of simple pill-box cavities. For each cavity the formulae of the diffractional model (12)-(15) are valid. So the loss factor of the arbitrary form taper can be valuated analytically. The numerical simulations with the code TBCI are in agreement with analytical calculations.

2. It is possible to use these analytical formulae for the optimization of a taper profile for decreasing the loss factor. Some examples are given.

3. For the DAΦNE parameters the interference of waves diffracted in different elements of the vacuum chamber is small. The loss factor of the cavity proposed for the main ring [2] is the sum of the loss factors of the cavity itself and the symmetric taper. It is also possible to approximate the loss factor of a taper with a complicated boundary by the sum of loss factors of linear tapers.

4. The combination "cavity - taper" proposed for the DAΦNE main ring [2] is optimal from the point of view of minimization of the loss factor. Additional decreasing in loss factor may be reached by changing the form of the cavity. But it is not reasonable because of shunt impedance decreasing. Variation of the taper profile does not give any significant improvement in the loss factor.

References

[1] S. A. Heifets, "Diffractive model of the high frequency impedance", Physical review D, V. 40, No. 9, p. 3097-3106.

[2] S. Bartalucci, L. Palumbo and B. Spataro, "A low loss cavity for DAΦNE main ring", DAΦNE Techn. Note, G-6 (1991).

[3] S. A. Heifets, S. A. Kheifets, "Coupling impedance in modern accelerators", SLAC-PUB-5297, September 1990(A).

APPENDIX

The impedance of the tapered cavity $Z_l^T(k)$ presented in Fig. 8

$$Z_l^T(k) = Z_i^T(a,b,L+2g1) + \delta Z_l^T \qquad (A.1)$$

The first term in (A.1) is the impedance of the cavity with the radius \`b\`, the length L + 2g1 with two tapers of the length g2 with the beam pipe radius \`a\`. The second term gives a correction due to the upper part (z >b) of the whole tapered cavity.

The real part of the impedance Z_i^T (a, b, L + 2g1) takes the form:

$$R_eZ_i^T = -\frac{Z_0 k}{2\pi}\int_a^b dr' \int_{-k}^{+k} \frac{dq}{k-q} J_0(as)\frac{\partial J_0(r's)}{\partial r'}[\sin^2(\frac{k-q}{2}(r'-a)\cot\alpha_1) -$$

$$-\sin^2(\frac{k-q}{2}(L+2(g1+g2)-(r'-a)\cot\alpha_1))] \qquad (A.2)$$

It was shown [3] that ReZ_i^T can be approximated by:

$$R_eZ_i^T = \frac{2}{c}\int_{-k}^{+k} \frac{dq}{k-q} J_0(as)[J_0(as)-J_0(bs)]\sin^2(\frac{L+2(g1+g2)}{2}(k-q))-$$

$$-\frac{2}{c}\int_{-k}^{+k} \frac{dq}{k-q} J_0(as)[J_0(as)-J_0(bs)]\sin^2(\frac{g2}{2}(k-q)) \qquad (A.3)$$

The first term is the real part of the impedance of the nontapered cavity with the length L+2g1+2g2,the radius \`b\` and the beam pipe radius \`a\`. The second term is the real part of the impedance of the nontapered cavity with the length g2,the radius \`b\` and the beam pipe radius \`a\`.

The real part of δZ_l^T has the form:

$$R_e\delta Z_l^T = -\frac{Z_0 k}{2\pi}\int_b^d dr' \int_{+k}^{-k} \frac{dq}{k-q} J_0(as)\frac{\partial J_0(r's)}{\partial r'}[\sin^2(\frac{k-q}{2}(r'-b)\cot\alpha_2) -$$

$$-\sin^2(\frac{k-q}{2}(L+2g1-(r'-b)\cot\alpha_2))] \qquad (A.4)$$

With the same accuracy as in (A.3) we can write:

$$R_e\delta Z_l^T = R_e\delta Z_{l_1}^T - R_e\delta Z_{l_2}^T \qquad \text{(A.5)}$$

$$R_e\delta Z_{l_1}^T = \frac{2}{c}\int_{-k}^{+k}\frac{dq}{k-q}J_0(as)[J_0(bs)-J_0(ds)]\sin^2(\frac{L+2g1}{2}(k-q)) \qquad \text{(A.6)}$$

$$R_e\delta Z_{l_2}^T = \frac{2}{c}\int_{-k}^{+k}\frac{dq}{k-q}J_0(as)[J_0(bs)-J_0(ds)]\sin^2(\frac{g1}{2}(k-q)) \qquad \text{(A.7)}$$

Then we will use the property that we have used in (10):

$$J_0(as)[J_0(bs) - J_0(ds)] = J_0(as)[J_0(as) - J_0(ds)] - J_0(as)[J_0(as) - J_0(bs)] \qquad \text{(A.8)}$$

$$R_e\delta Z_{l_1}^T = \frac{2}{c}\int_{-k}^{+k}\frac{dq}{k-q}J_0(as)[J_0(as)-J_0(ds)]\sin^2(\frac{L+2g1}{2}(k-q)) -$$

$$- \frac{2}{c}\int_{-k}^{+k}\frac{dq}{k-q}J_0(as)[J_0(as)-J_0(bs)]\sin^2(\frac{L+2g1}{2}(k-q)) \qquad \text{(A.9)}$$

$$R_e\delta Z_{l_2}^T = \frac{2}{c}\int_{-k}^{+k}\frac{dq}{k-q}J_0(as)[J_0(as)-J_0(ds)]\sin^2(\frac{g1}{2}(k-q))-$$

$$- \frac{2}{c}\int_{-k}^{+k}\frac{dq}{k-q}J_0(as)[J_0(as)-J_0(bs)]\sin^2(\frac{g1}{2}(k-q)) \qquad \text{(A.10)}$$

According to (A.3) the difference between the first terms in (A.9) and (A.10) gives the real part of the impedance of tapered cavity with the length L, the radius \`d\` with the taper of the length g1 and the beam pipe radius \`a\`. The difference between the second terms in (A.9) and (A.10) gives the real part of the impedance of tapered cavity with the length L,the radius \`b\`,the tapers of the length g1 and beam pipe radius \`a\`.

So, the loss factor k_l^T of the tapered cavity shown in Fig. 8 is:

$$k_l^T = k_{l_1}^T + k_{l_2}^T - k_{l_3}^T \qquad \text{(A.11)}$$

where k_{li}^T corresponds to i - tapered cavity presented in Fig. 9.