DAΦNE TECHNICAL NOTE

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LANDAU DAMPING OF LONGITUDINAL MULTI-BUNCH INSTABILITIES IN DAΦNE

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Introduction

One of the main problems in the beam dynamics in DAΦNE are the multibunch instabilities caused by the strong coupling between the beam and parasitic HOMs of the RF cavity [1]. Strong efforts have been done to optimize the RF cavity shape [2] and to apply different HOM damping techniques in order to reduce the growth rates of the instabilities [3].

In spite of the satisfactory results in HOM damping, due to the high beam current in DAΦNE, a feedback system or another damping mechanism is still needed to kill the residual coherent oscillations. A digital bunch-by-bunch feedback system [4] is planned to be installed in the DAΦNE main rings to fight the dipole (rigid motion) coupled-bunch longitudinal instability.

As it has been shown in [1], the longitudinal quadrupole modes (and, probably, higher modes) can be also dangerous if special measures to damp them are not undertaken, while at the moment no feedback system is foreseen to deal with the quadrupole (and higher) modes. So, to avoid using a feedback system we have to investigate whether another damping mechanism exists to cope with the quadrupole mode instability.

Landau damping is a possible candidate for this purpose. In the following we consider Landau damping of both the dipole and quadrupole modes due to nonlinearities of the RF fields and discuss the necessary conditions to keep bunch motion stable.

Stability diagrams

We assume that a particle distribution function may be represented as the sum of a stationary distribution $g_0(\hat{\tau})$ and a coherent perturbation:

$$\Psi(\psi_0, \hat{\tau}, t) = g_0(\hat{\tau}) + \sum_m g_m(\hat{\tau})\, e^{-j\, m\, \psi_0}\, e^{j(\Omega_m - m\,\omega_s)t} \qquad (1)$$

where $\hat{\tau}$ and ψ_0 are polar amplitude and phase coordinates in the longitudinal phase space; ω_s is the angular incoherent synchrotron frequency; Ω_m is the angular frequency of the m^{th} coherent mode.

Then the linearized Vlasov equation gives the following equation of coherent motion for the m^{th} mode [5]:

$$j^{1-m}[\,\Omega_m - m\,\omega_s(\hat{\tau})\,]\,\hat{\tau}\,g_m(\hat{\tau}) = -\frac{m\,\alpha\,I}{\omega_s(\hat{\tau})\,(E/e)}\frac{\partial g_0}{\partial\hat{\tau}}\sum_p \frac{Z_{//}(p)}{p}\,J_m(p\,\omega_0\,\hat{\tau})\,\sigma_m(p) \qquad (2)$$

Here $\sigma_m(p)$ is the spectrum amplitude of m^{th} mode at frequency $\omega_p = p\omega_0 + \Omega_m$ with the index $p = Mk + n$, where M is the number of bunches in a beam, k the revolution harmonic, and n the coupled bunch mode number:

$$\sigma_m(p) = j^{-m}\int_0^{\infty} J_m(\,p\,\omega_0\,\hat{\tau}\,)\,g_m(\hat{\tau})\,\hat{\tau}\,d\hat{\tau} \qquad (3)$$

$Z_{//}(p)$ is the longitudinal impedance due to higher-order modes at frequencies ω_p; J_m is the Bessel function; α the momentum compaction; I the average beam current; E the energy of particles.

The dependence of the synchrotron frequency on the amplitude of synchrotron oscillation $\hat{\tau}$ appears due to the nonlinearity of the RF field:

$$\omega_s(\hat{\tau}) = \omega_{s0}\left[\,1 - \left(\frac{h\,\omega_0}{4}\right)^2\hat{\tau}^2\,\right] \qquad (4)$$

Two comments should be made on eq. (2):

- Here we consider only "the most coherent modes" when the azimuthal mode number is equal to the radial one. These modes have the shortest rise times.
- Eq. (2) implies that the coherent frequency shift $\Delta\Omega = \Omega_m - m\,\omega_s$ is small compared to the angular synchrotron frequency ω_s. This allows to neglect the coupling between modes with different m.

Multiplying both sides of eq. (2) by

$$\frac{J_m(\,l\,\omega_0\,\hat{\tau}\,)}{\Omega_m - m\,\omega_s(\hat{\tau})}$$

and integrating from 0 to ∞ we arrive to the dispersion relation:

$$-m\int_0^{\infty} J_m(l\,\omega_0\,\hat{\tau})g_m(\hat{\tau})\,\hat{\tau}\,d\hat{\tau} = j\,\frac{m\,\alpha\,I}{\omega_{s0}\,(E/e)}\sum_p\frac{Z_{//}(p)}{p}\;\sigma_m(p)\int_0^{\infty}\frac{\partial g_0}{\partial\hat{\tau}}\frac{J_m(p\,\omega_0\,\hat{\tau})\,J_m(l\,\omega_0\,\hat{\tau})}{[\,\Omega_m - m\,\omega_s(\hat{\tau})\,]}d\hat{\tau} \qquad (5)$$

As far as for our set of parameters (see Table 1 below) the synchrotron frequency spread due to RF field nonlinearities is small compared to the angular synchrotron frequency ω_{so} and $\Delta\Omega << \omega_s$ we used the following approximation in getting (5):

$$\omega_s(\hat{\tau})\,[\,\Omega_m - m\,\omega_s(\hat{\tau})\,] = \omega_{s0}\,[\,\Omega_m - m\,\omega_s(\hat{\tau})\,]$$

Let us assume that the coherent motion is driven by a narrow band resonant impedance and the only line of the bunch spectrum lies within this narrow frequency band. Then, the sum over p is eliminated and putting p = l we reduce the dispersion relation to the form:

$$1 = j \frac{m \alpha I}{\omega_{s0} (E/e)} \frac{Z_{//}(p)}{p} \int_0^\infty \frac{\partial g_0}{\partial \hat{\tau}} \frac{J_m^2 (p \omega_0 \hat{\tau})}{[\Omega_m - m \omega_s(\hat{\tau})]} d\hat{\tau} \tag{6}$$

We consider a stationary Gaussian amplitude density distribution:

$$g_0(\hat{\tau}) = \frac{1}{2 \pi \sigma_\tau^2} \exp\left\{ -\frac{1}{2} \left(\frac{\hat{\tau}}{\sigma_\tau}\right)^2 \right\} \tag{7}$$

Then, we rewrite the dispersion relation (6) taking into account (4) and (7):

$$1 = -j \frac{4 \alpha I}{\pi \omega_{s0}^2 (E/e) (h \omega_0)^2 \sigma_\tau^4} \frac{Z_{//}(p)}{p} \int_0^\infty \frac{\exp(-x) J_m^2 (p \omega_0 \sigma_\tau \sqrt{2x})}{[x - y]} dx \tag{8}$$

where

$$y = -\frac{8 (\Omega_m - m \omega_{so})}{m \omega_{so} (h \omega_0 \sigma_\tau)^2} \tag{9}$$

It is convenient to present the dispersion relation (8) in terms of the imaginary and real part of the longitudinal impedance. For the dipole mode (m = 1) we have:

$$\mathrm{Im}\{Z_{//}\} = \frac{\pi (h \omega_{s0} \sigma_\tau)^2 (E/e)}{2 p \alpha I} \mathrm{Re}\{G_1(y)\} \tag{10}$$

$$\mathrm{Re}\{Z_{//}\} = -\frac{\pi (h \omega_{s0} \sigma_\tau)^2 (E/e)}{2 p \alpha I} \mathrm{Im}\{G_1(y)\} \tag{11}$$

where we define $G_1(y)$ as:

$$G_1^{-1}(y) = \frac{2}{(p \omega_0 \sigma_\tau)^2} \int_0^\infty \frac{\exp(-x) J_1^2 (p \omega_0 \sigma_\tau \sqrt{2x})}{[x - y]} dx \tag{12}$$

The stability limit is found by imposing Im(y) --> 0^-. The threshold curve of the instability is given by the curve traced out by $G_1(y)$ in the complex plane (See Fig. 1).

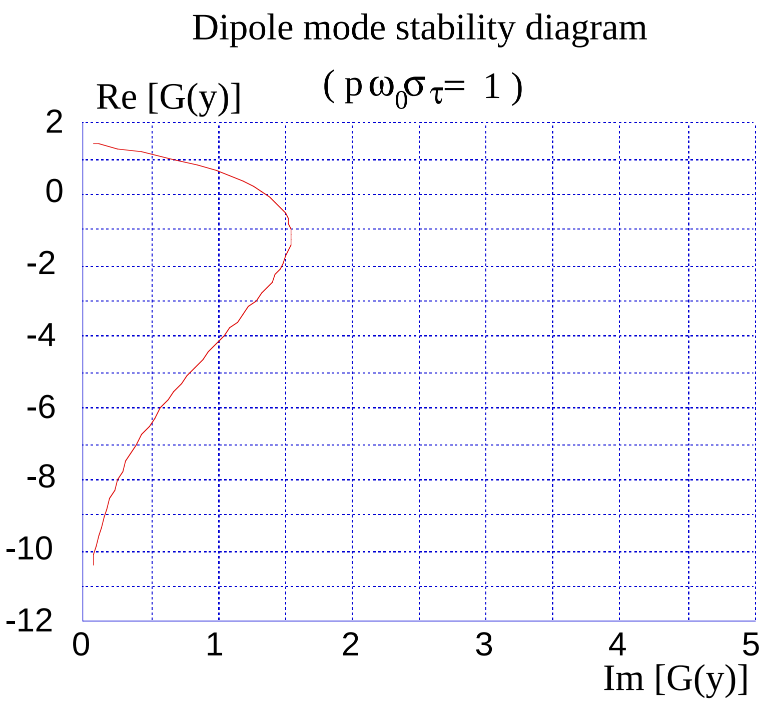

Fig. 1 - Stability diagram for longitudinal dipole mode at $p\omega_0\sigma_\tau = 1$.

It worth noting that for very short bunches with $p\omega_0\sigma_\tau << 1$ we can expand Bessel function in (12) in series keeping only the first term of the expansion:

$$J_1(z) \sim z/2 \qquad \text{for } z << 1 \tag{13}$$

Then we will get exactly Wang's dispersion relation for short bunches [6].

In the case of full coupling with a higher-order mode, Im{Z} is equal to 0 and Re{Z}= R_s, where R_s is the shunt impedance of the mode. So, the limit on R_s is defined by intersection of the curve $G_1(y)$ with the axis Re{G(y)} = 0.

$$R_s^{max} = -\frac{\pi\,(h\,\omega_{s0}\,\sigma_\tau)^2\,(E/e)}{2\,p\,\alpha\,I}\,\mathrm{Im}\{G_1(y)\}\big|_{\mathrm{Re}\{G_1(y)\}=0} \tag{14}$$

This means that the condition $R_s < R_s^{max}$ must be fulfilled to keep bunch motion stable. It should be noticed that R_s^{max} does not depend on the momentum compaction, while it is inversely proportional to the beam current. The dependence of R_s^{max} on the bunch length is hidden in the product $\sigma_\tau^2 \mathrm{Im}\{G\}|_{\mathrm{Re}\{G\}=0}$ and on the frequency - in $\mathrm{Im}\{G\}|_{\mathrm{Re}\{G\}=0}/p$. Fig. 2 shows the dependence of $\mathrm{Im}\{G\}|_{\mathrm{Re}\{G\}=0}$ on the parameter $p\omega_0\sigma_\tau$.

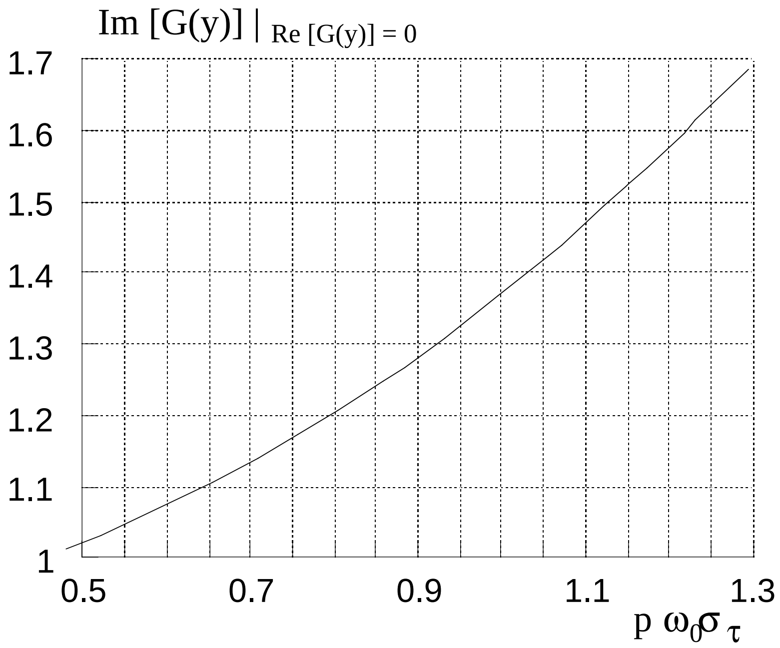

Fig. 2 - Dependence of $Im\{G\}|_{Re\{G\}=0}$ *on the parameter* $p\omega_o\sigma_\tau$.

For the quadrupole mode (m = 2) we can rewrite the dispersion relation (8) in the following form:

$$\mathrm{Im}\,\{Z_{//}\} = \frac{4\pi\,(h\,\omega_{s0}\,)^2\,(E/e)}{\omega_0^2\,p^3\,\alpha\,I}\,\mathrm{Re}\{G_2(y)\} \tag{15}$$

$$\mathrm{Re}\,\{Z_{//}\} = -\,\frac{4\pi\,(h\,\omega_{s0}\,)^2\,(E/e)}{\omega_0^2\,p^3\,\alpha\,I}\,\mathrm{Im}\{G_2(y)\} \tag{16}$$

where

$$G_2^{-1}(y) = \frac{16}{(p\,\omega_0\,\sigma_\tau)^4}\int_0^\infty \frac{\exp\,(\,-\,x\,)J_2^2\,(p\,\omega_0\,\sigma_\tau\,\sqrt{2\,x}\,)}{[\,x - y\,]}dx \tag{17}$$

For very short bunches $p\omega_o\sigma_\tau << 1$ we can use an approximation:

$$J_2(z) \sim \left(\frac{z}{2}\right)^2 \frac{1}{\Gamma(3)} = \frac{z^2}{8} \qquad \text{for } z << 1 \tag{18}$$

and it is easy to see that eqs. (15)-(17) are transformed into Wang's dispersion relation for short bunches [6].

The stability boundary curve for the quadrupole mode is shown in Fig. 3.

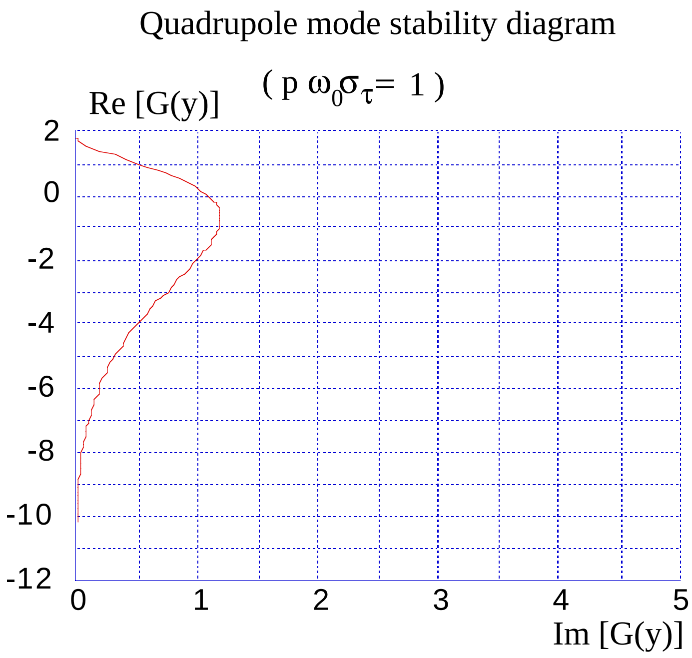

Fig. 3 - Stability diagram for longitudinal quadrupole mode at $p\omega_o\sigma_\tau$=1.

In the case of full coupling R_s must be less that $R_s{}^{max}$ to keep quadrupole motion stable:

$$R_s^{max} = -\frac{4\pi\,(h\,\omega_{s0})^2\,(E/e)}{\omega_0^2\,p^3\,\alpha\,I}\,\mathrm{Im}\{G_2(y)\}\big|_{\mathrm{Re}\{G_2(y)\}=0} \qquad (19)$$

We should mention here that the stability limit has much stronger dependence on frequency and weaker dependence on the bunch length (only through $G_2(y)$) than that found for the dipole motion (See eq. (14) for comparison). Fig. 4 presents $\mathrm{Im}\{G_2\}|_{\mathrm{Re}\{G2\}=0}$ as a function of $p\omega_o\sigma_\tau$.

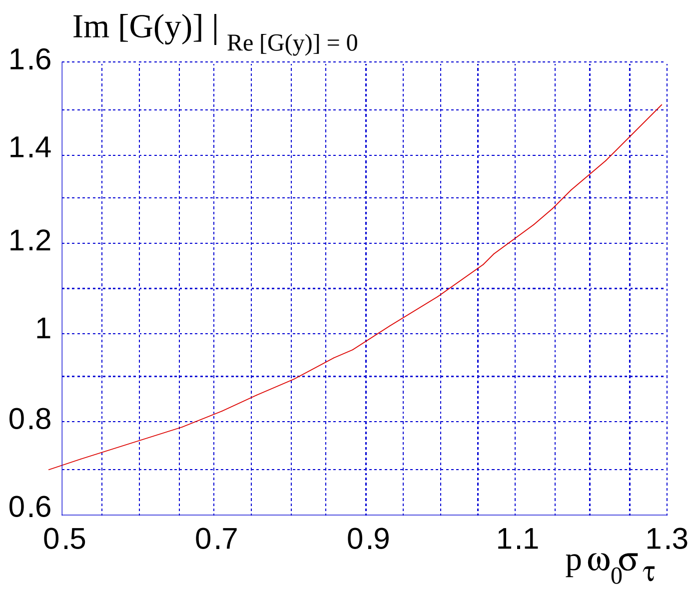

Fig. 4 - $\mathrm{Im}\{G_2\}|_{\mathrm{Re}\{G2\}=0}$ as a function of $p\omega_o\sigma_\tau$.

Stability limits

In this section we apply the stability diagrams to find stability limits on Q for the case of full coupling. We assume that the ratio R/Q is kept constant.

Table1 gives the DAΦNE single ring parameters which we used in our calculations. If the parameters change, eqs. (14), (19) (and Fig. 2 and Fig. 4) can be applied as a scaling to find stability limits on the shunt impedance of HOMs.

Table 1 - DAΦNE parameters used in calculations

Revolution frequency, MHz	3.069
Harmonic number	120
RF frequency, MHz	368.25
Energy, MeV	510
Momentum compaction	0.005846
Synchrotron frequency, kHz	22.877
Bunch length, cm	3
Average current/bunch, mA	43.75
Number of bunches	30

Table 2 gives calculated parameters of HOM modes of the DAΦNE RF cavity and rise times of the dipole mode. Table 2 is mostly drawn from [1] except for the last column where we give values of quality factors which it is necessary to reach by damping techniques to keep coherent dipole oscillation stabilized by Landau damping.

Table 2 - Rise times of dipole modes and Q's required to be Landau damped

f, MHz	R/Q	Q_0	τ	Q_{damped}
734.42	13.5	52000	11 µs	97
797.89	0.02	85000	763 µs	61680
1006.20	0.004	68000	5 ms	-------
1086.44	0.11	69000	219 µs	9510
1163.18	0.24	61000	119 µs	4160
1234.49	1.49	74000	29 µs	625
1317.27	0.33	60000	90 µs	2740
1357.49	1.37	75000	29 µs	650
1429.06	0.73	63000	48 µs	1190
1527.46	0.06	66000	363 µs	14210
1529.25	1.88	62000	25 µs	450
1619.86	0.82	74000	41 µs	1020
1643.96	1.28	68000	30 µs	650
1725.08	0.72	79000	45 µs	1145
1762.63	2.25	70000	21 µs	365
1767.80	0.82	73000	41 µs	1000
1800.69	0.22	76000	119 µs	3720
1869.62	0.10	76000	230 µs	8140
1890.12	0.73	64000	50 µs	1115
1978.98	0.23	78000	115 µs	3520
1985.50	0.18	96000	122 µs	4490

Table 3 gives calculated parameters of HOM modes of the DAΦNE RF cavity and rise times of the quadrupole modes. In the last column we give the limiting values of Q. Below these values the quadrupole mode coupled-bunch longitudinal instability is damped.

Table 3 - Rise times of quadrupole modes and Q's required to be Landau damped

f, MHz	R/Q	Q_0	τ	Q_{damped}
734.42	13.5	52000	43 μs	2500
797.89	0.02	85000	5.6 ms	-------
1006.20	0.004	68000	29 ms	-------
1086.44	0.11	69000	827 μs	-------
1163.18	0.24	61000	360 μs	44090
1234.49	1.49	74000	65 μs	6900
1317.27	0.33	60000	237 μs	24240
1357.49	1.37	75000	61 μs	5470
1429.06	0.73	63000	104 μs	9180
1527.46	0.06	66000	800 μs	-------
1529.25	1.88	62000	43 μs	3120
1619.86	0.82	74000	70 μs	6390
1643.96	1.28	68000	50 μs	3980
1725.08	0.72	79000	68 μs	6540
1762.63	2.25	70000	30 μs	2000
1767.80	0.82	73000	63 μs	5470
1800.69	0.22	76000	192 μs	19760
1869.62	0.10	76000	349 μs	40950
1890.12	0.73	64000	72 μs	5500
1978.98	0.23	78000	162 μs	16360
1985.50	0.18	96000	165 μs	20706

Below, for the comparison with the analytical results, we reproduce the table of experimental results obtained by equipping the test cavity with 5 waveguides and ferrite loads [3] for some highest HOM:

Table 4 - Cavity model test result (Monopoles)

Mode	f, MHz	Q_0	R/Q, Ω	Q_{damped}	R_{sh}, kΩ
0-MM1	747.5	24000	16	60	0.96
0-EM2	796.8	40000	0.5	270	0.13
0-MM2	1023.6	28000	0.9	------	---------
0-EM3	1121.1	12000	0.3	600	0.21
0-MM3	1175.9	5000	0.6	140	0.08
0-EM4	1201.5	9000	0.2	110	0.02
0-EM5	1369.0	5000	2.0	300	0.6
0-MM4	1431.7	2000	1.0	150	0.15
0-EM6	1465.0	2000	0.1	200	0.02

Conclusions

We can conclude from comparison of analytical results (Tables 2, 3) and experimental results (Table 4) that:

- Obtained experimental damped Q values are well below the stability limit values for quadrupole modes. So we can expect that quadrupole coherent oscillations are to be stable and no additional feedback system is necessary.

- For the dipole modes experimental value are close to those obtained analytically. Keeping in mind that the stability limits scales approximately as $\sigma^{2.4}$ for the range of interest of σ and can be much lower for shorter bunches we see that Landau damping can not fight the dipole mode instability alone.

References

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