DAΦNE TECHNICAL NOTE

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STUDY OF MICROWAVE INSTABILITY FOR DAΦNE ACCUMULATOR RING

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1 Introduction

The DAΦNE Φ-Factory is an e^+ e^- high luminosity collider under construction at the INFN-LNF (1).

At present its accumulator ring is being commissioned, and the first beam has been successfully stored (2). Measurements of bunch lengthening and energy spread have not been performed yet, and therefore in this paper we present the results of a theoretical-numerical study.

The preliminary estimates of the beam impedance (3) and the microwave instability threshold have shown that the nominal current is far above the turbulence threshold.

In order to study the bunch lengthening process and the microwave instability manifestation in the accumulator ring, we use a numerical tracking code that simulates the single bunch longitudinal dynamics. We also undertake an analytical study and compare different models that estimate the instability threshold with the tracking code results.

In Section 2 we describe the numerical results obtained with the multiparticle tracking code, while in Section 3 we perform the analytical study of the mode coupling leading to turbulence.

Table 1 shows the accumulator ring parameters relevant for both the numerical and analytical studies.

Table I. DAΦNE Accumulator Parameter List

Nominal Energy	MeV	510
Machine Length	m	32.56
Momentum Compaction		.034
Damping Time	ms	10.71
Natural Rms Bunch Length	cm	1.75
Natural Rms Energy Spread		4.1×10^{-4}
RF Peak Voltage	KV	200
Harmonic Number		8
Maximum Number of Particles		9×10^{10}

2 Numerical Simulations of the Bunch Lengthening

In order to study the bunch lengthening in the DAΦNE accumulator ring, we use a standard numerical tracking technique which has already been successfully applied in the bunch lengthening simulations for the SLC damping rings (4), SPEAR (5), PETRA and LEP (6).

The motion of N_s super particles, representing a bunch with a total charge Q, is described in the longitudinal phase space by:

$$\begin{aligned} \varepsilon_i(n) &= \varepsilon_i(n-1) - \frac{2T_o}{\tau_\varepsilon}\varepsilon_i(n-1) + 2\sigma_{\varepsilon o}\sqrt{\frac{T_o}{\tau_\varepsilon}}R_i(n) + V'_{rf}z_i(n) + V_{ind}(z_i)\ , \\ z_i(n) &= z_i(n-1) + \frac{\alpha_c L_o}{E}\varepsilon_i(n)\ . \end{aligned} \tag{1}$$

In the equation (1) $\varepsilon_i(n)$ and $z_i(n)$ are respectively the energy and the position coordinates of the ith particle after n revolutions in the storage ring; T_o is the revolution period; τ_ε the damping time; $\sigma_{\varepsilon o}$ the rms of the bunch unperturbed energy distribution; V'_{rf} the derivative of the RF voltage with respect to the particle longitudinal coordinate; α_c the momentum compaction; L_o the machine length; E the nominal energy, $V_{ind}(z_i)$ the voltage induced by the whole bunch and seen by the ith particle in the position z_i, and $R_i(n)$ a random number obtained from a normally distributed function with mean 0 and rms 1.

At each turn, all the superparticles are distributed in a given number of bins in accordance with their longitudinal position. The bunch is considered as a composition of short gaussian bunches (substantially shorter than the bunch itself) and located at the bin centers. The induced voltage is calculated as a convolution of the wake potentials of the short gaussian bunches.

For DAΦNE accumulator ring, the wake potential of a gaussian bunch with an rms bunch length of $\sigma_z = 5$ mm is used in the induced voltage calculations.

In order to determine the wake potentials of the bunch in all the important vacuum chamber components, we used ABCI (7) and MAFIA (8) computer codes. The dominant impedance elements were found to be the injection-extraction kickers, the RF cavity and numerous small discontinuities. The resulting total wake potential is shown in Fig. 1.

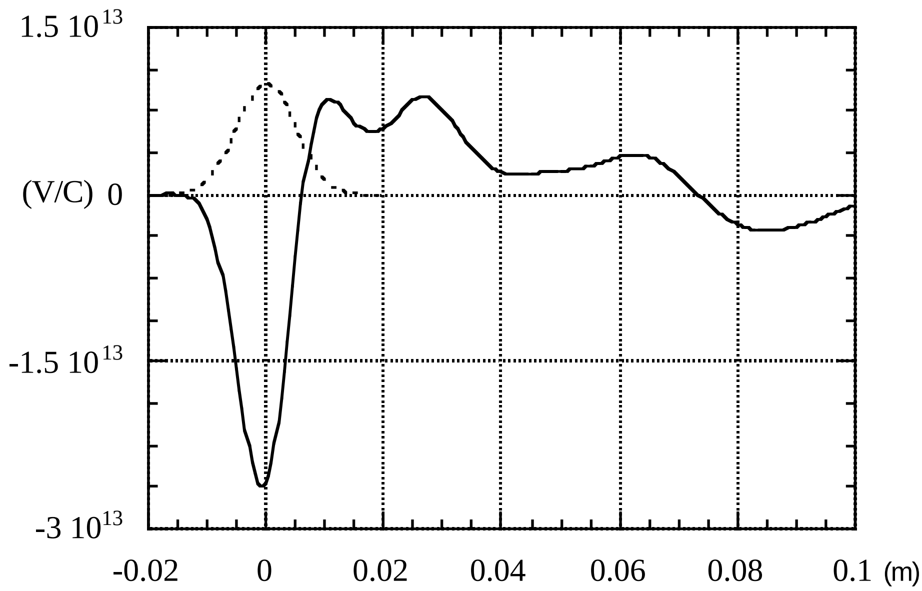

FIGURE 1. DAΦNE accumulator wake potential of 5 mm gaussian bunch.

The main results of the simulations are shown in Figs. 2 - 3.

Figure 2 represents the rms bunch length averaged over many revolutions as a function of number of particles in the bunch. The behavior is very regular and it is impossible to distinguish any instability threshold. However, the threshold is quite evident in Fig. 3 where the average rms relative energy spread versus the number of particles N is shown. The microwave instability threshold is clearly seen at $N_{th} = 2 \times 10^{10}$.

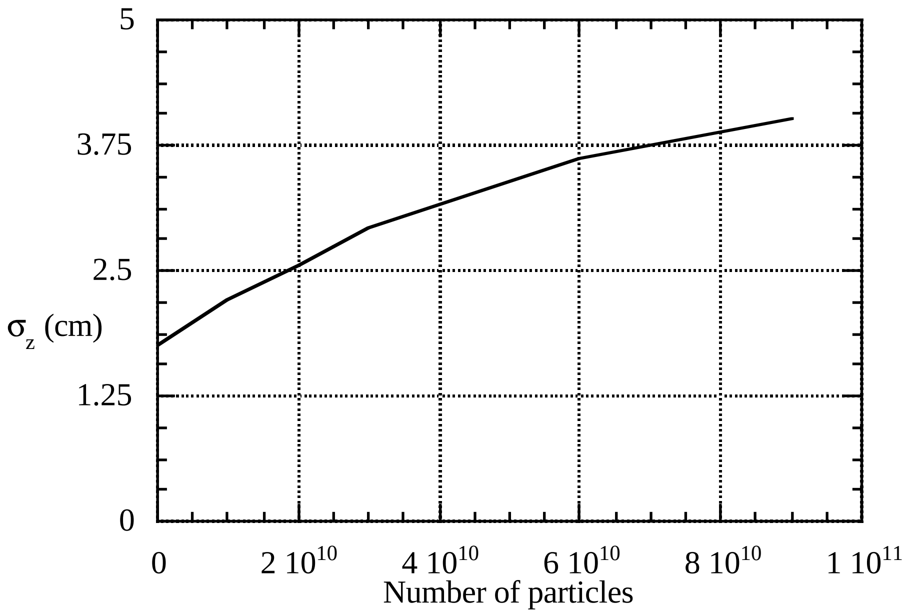

FIGURE 2. Rms bunch length versus number of particles.

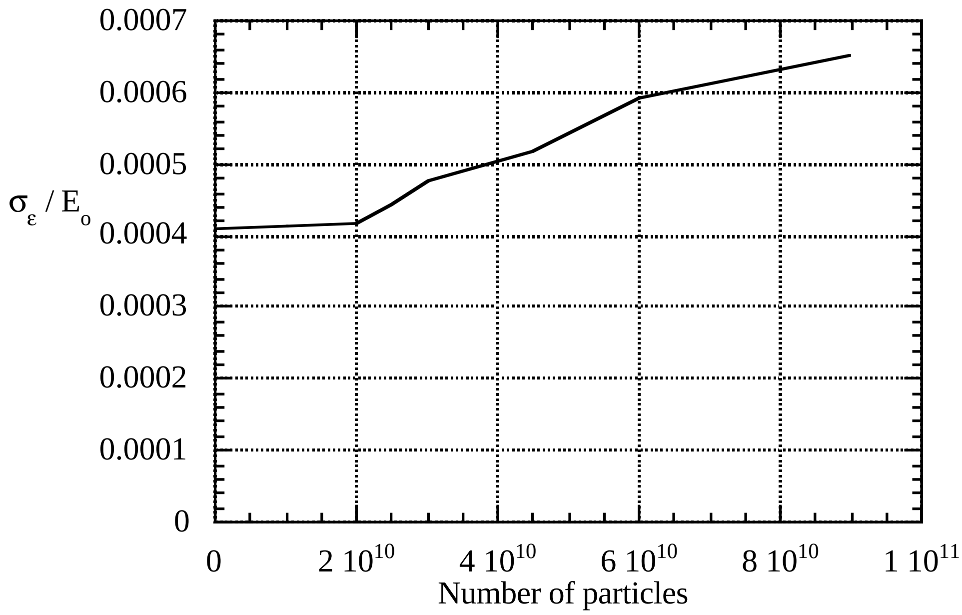

FIGURE 3. Rms energy spread versus number of particles.

Since the accumulator ring has to operate above the predicted threshold, we investigated the turn-by-turn rms bunch length behavior at the nominal current, with $N = 9 \times 10^{10}$. Figure 4 displays the turn-by-turn bunch length after 6 damping time. The maximum variation in σ_z is less than 4% and it is considered not dangerous for the machine operation.

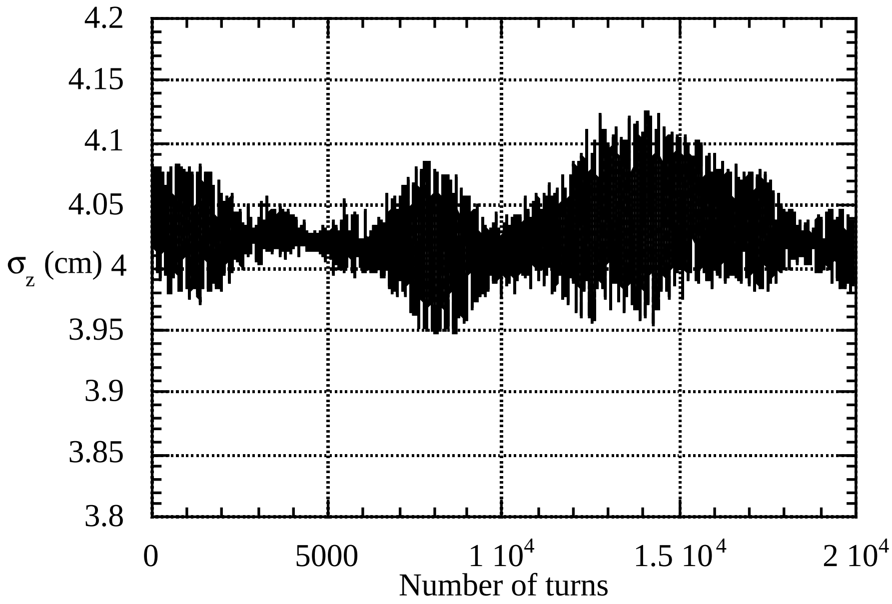

FIGURE 4. Rms bunch length as function of turns with τ_ε = 10.71 ms.

It is worth to mention the importance of simulations with the real damping time, even if it requires vast amount of computing time. As an example, Figure 5 shows the bunch length versus number of turns, but with τ_ε reduced by a factor of 20. From the plot, wrong conclusions may be drawn, like to see nonexistent saw-tooth behavior.

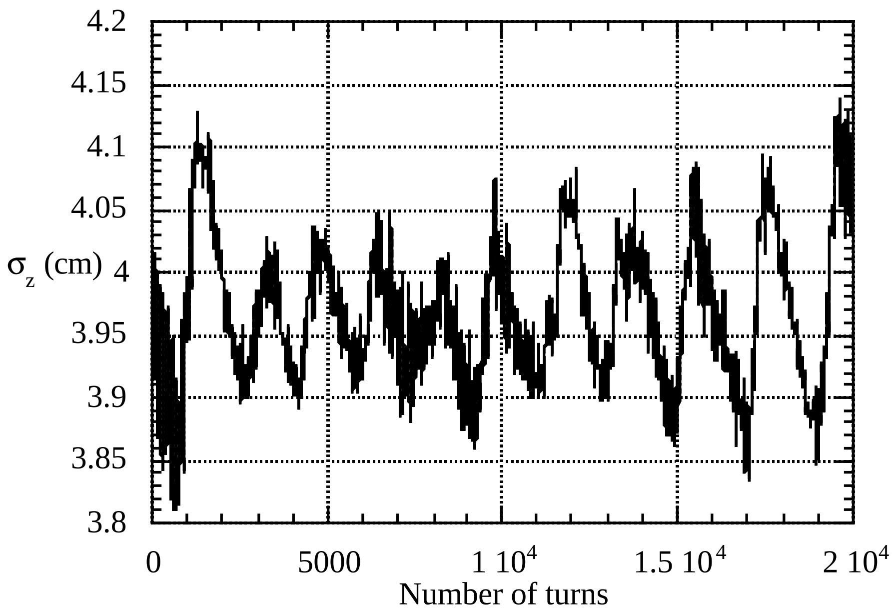

FIGURE 5. Rms bunch length as function of turns with τ_ε = .5 ms.

To conclude this section, in Fig. 6 we compare the bunch shape in the turbulent regime ($N = 9 \times 10^{10}$ particles) with a gaussian bunch of the same rms length. The shape is more bulbous than gaussian and slightly distorted, meaning that the ring wake field has a mainly inductive component. From the figure we can also conclude that the gaussian distribution approximates very well the bunch shape even in the strong turbulent regime.

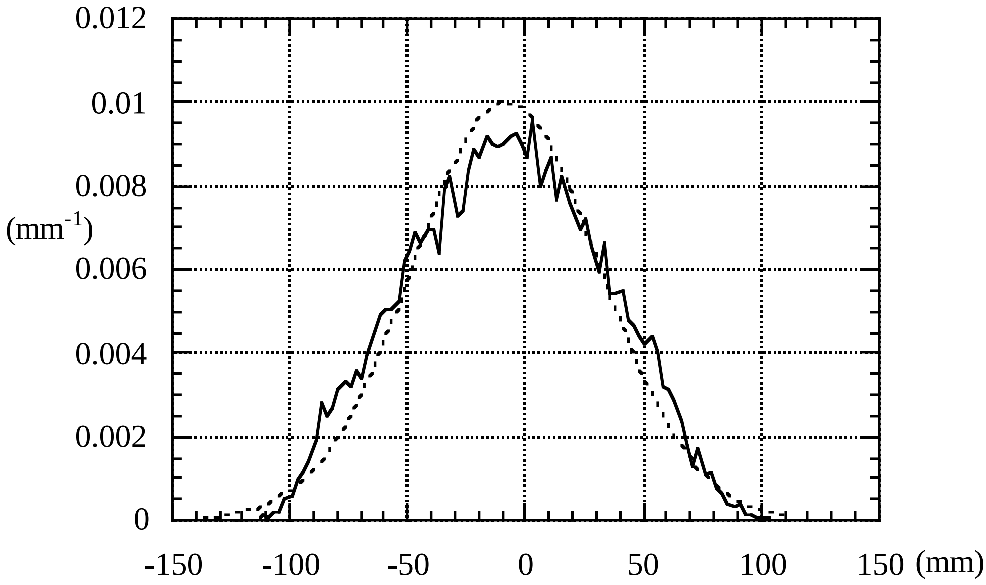

FIGURE 6. Bunch shape and gaussian distribution.

3 Analytical Study

The microwave instability regime is usually described by a mode coupling theory. If the bunch distribution has a head-tail symmetry, that is the wake field is mainly inductive, the azimuthal modes of the distribution function can couple after the mode frequencies have been shifted by large amounts (comparable to the synchrotron frequency). In this situation we have the so-called strong microwave instability characterized by a growth rate of the same order of magnitude of the synchrotron frequency.

When the wake field has a high resistive component, there is a head-tail asymmetry and radial modes can trigger the instability. The mode frequency shifts are small, the coupling is weak and can be stabilized by both the radiation damping and Landau damping effects.

Before analyzing the microwave instability, we start to study the bunch length below the instability threshold. We compare solutions of the stationary self-consistent Fokker-Planck (9) equation, or Haissinski equation, with the results of the tracking code. The difference is no more than few percent, showing that the Haissinski equation is thus a valid tool to evaluate the bunch length in the steady state regime.

As far as the microwave threshold is concerned, the simplest approach is to use the Boussard criterion (10):

$$N_{th} = \frac{(2\pi)^{3/2}\alpha_c \sigma_\varepsilon^2 (E/e)}{ce}\frac{\sigma_z}{|Z/n|}, \tag{2}$$

where the impedance $|Z/n|$ is calculated from the wake field using a Broad-Band Resonator model. For DAΦNE accumulator ring, the value N_{th} given by equation (2) has been found to be $N_{th} = 6.25 \times 10^9$, that is by a factor of three smaller than the one obtained with the simulations. This means that more rigorous analysis is necessary to explain the simulation results.

To further investigate the single bunch behavior in the turbulent regime, we use the mode coupling theory based on the Vlasov equation:

$$\frac{\partial\psi(z,\varepsilon;t)}{\partial t} = -c\frac{\partial\psi(z,\varepsilon;t)}{\partial\varepsilon}\frac{\partial H(z,\varepsilon;t)}{\partial z} + c\frac{\partial\psi(z,\varepsilon;t)}{\partial z}\frac{\partial H(z,\varepsilon;t)}{\partial\varepsilon}, \tag{3}$$

with $\psi(z,\varepsilon;t)$ the distribution function, and $H(z,\varepsilon;t)$ the single particle Hamiltonian. By linearizing the function $\psi(z,\varepsilon;t)$ around the stationary distribution and with the azimuthal mode expansion of the perturbed distribution function, we obtain for each azimuthal number m the general equation (11):

$$\begin{aligned}[\Omega - m\omega(J)]R_m(J) = -i\frac{\alpha_c c e^2 N}{4\pi^2\omega(J)ET_o}\frac{\partial\psi_o(J)}{\partial J}\int_0^{2\pi}d\phi\int_0^{2\pi}d\phi'\int_{-\infty}^{\infty}d\omega\int_0^{\infty}dJ' \\ R_l(J')\varepsilon(J,\phi)e^{i(m\phi-l\phi')}Z(\omega)e^{i\frac{\omega}{c}[z'(J'\phi')-z(J\phi)]}\end{aligned}, \tag{4}$$

where we have introduced the action and angle variables J and ϕ, with Ω the coherent oscillation frequency, $\omega(J)$ the synchrotron frequency depending on the oscillation amplitude, $R_m(J)$ the radial function of the m$^{\text{th}}$ azimuthal mode of the perturbed distribution, $\psi_o(J)$ the stationary distribution, and $Z(\omega)$ the longitudinal coupling impedance.

If we make the further assumption that the bunch is gaussian, hypothesis supported by Fig. 6 from which we see that the gaussian distribution approximates very well the bunch shape, equation (4) can be simplified to:

$$\begin{aligned}(\Omega - m\omega_{so})R_m(\hat z) = -i\frac{mce^2N}{T_o}\frac{1}{\hat z}\frac{\partial\psi_o(\hat z)}{\hat z}\sum_{l=-\infty}^{\infty} i^{(m-l)} \\ \int_{-\infty}^{\infty}\frac{Z(\omega)}{\omega}J_m\left(\frac{\omega}{c}\hat z\right)d\omega\int_0^{\infty}R_l(\hat z')J_l\left(\frac{\omega}{c}\hat z'\right)\hat z' d\hat z'\end{aligned}, \tag{5}$$

with ω_{so} the synchrotron frequency, $\hat z$ the single particle amplitude of oscillation and $J_m(x)$ the Bessel function of the first kind of m$^{\text{th}}$ order.

By expanding the radial function $R_m(\hat z)$ in terms of orthogonal polynomials, and considering only the most prominent radial mode, equation (5) can be transformed in a simple eigenvalue equation:

$$(\Omega - m\omega_{so})a_m = \sum_{l=-\infty}^{\infty} M^{ml}a_l\,, \tag{6}$$

to be satisfied for every mode number m, and with

$$M^{ml} = i\frac{\alpha_c c^2 e^2 N}{2\pi T_o E\omega_{so}\sigma_z^2}\frac{mi^{(m-l)}}{\sqrt{m!l!}}\int_{-\infty}^{\infty}\frac{Z(\omega)}{\omega}\left(\frac{\omega\sigma_z}{\sqrt{2}c}\right)^{m+l}e^{-\frac{\omega^2\sigma_z^2}{c^2}}d\omega\,. \tag{7}$$

The coherent frequencies Ω are therefore obtained by solving the equation:

$$\det\left[\mathbf{M}-\mathbf{I}(\Omega-m\omega_{so})\right]=0\,, \tag{8}$$

where $\mathbf{M}$ is the matrix given by equation (7), and $\mathbf{I}$ the identity matrix. The instability arises when the frequencies Ω become complex.

In order to calculate the microwave threshold, it is necessary to know the longitudinal coupling impedance as a function of the frequency ω. For DAΦNE accumulator ring the impedance is approximated reasonably well by (12)

$$Z(\omega)=R+i\omega L \tag{9}$$

In Fig. 7 we show a comparison between the wake field obtained with the simulations and the corresponding one obtained with the RL impedance.

Since inside the bunch distribution the two wakes do not manifest any significative difference, and the corresponding loss factor is the same in the two cases, we use the RL model to calculate the matrix elements. By substituting the relation (9) into equation (7), the integral can be solved giving:

$$M^{ml}=\frac{\alpha_c c^2 e^2 N}{2\sqrt{\pi}T_o E\omega_{so}\sigma_z^2}\frac{m i^{(m-l)}}{\sqrt{m!l!}2^{(m+l)}}\begin{cases} i\dfrac{Lc(m+l-1)!!}{\sigma_z} & (m+l)\text{ even} \\ \sqrt{2}R(m+l-2)!! & (m+l)\text{ odd}\end{cases}, \tag{10}$$

where

$$(2n+1)!!=1\cdot 3\cdot \mathrm{K}\ \cdot(2n+1)\,. \tag{11}$$

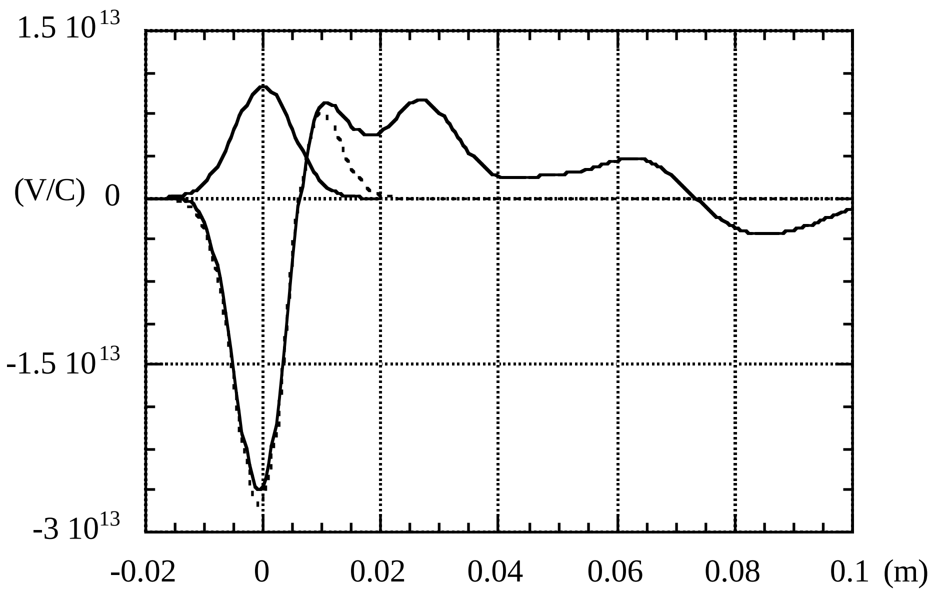

FIGURE 7. ABCI and RL impedance model wake fields.

With the matrix elements given by equation (10) we have solved equation (8) with the first six coherent azimuthal oscillation modes. The eigenfrequencies Ω are plotted in Fig. 8 versus the number of particles in a bunch.

The dashed line represents the imaginary part of the frequency, that is the growth rate. The microwave threshold is $N_{th} = 3\times10^{10}$, 1.5 times the one obtained with the simulations, and the instability is due to the coupling of the modes $m=1$ and $m=2$, dipole and quadrupole respectively.

For a complete analysis of the mode coupling, one should include in the treatment also the radial modes of oscillation for every azimuthal number. Unfortunately solutions for the Vlasov equation in this case can be found only when we consider a very simple distribution function, such as the so-called double water-bag distribution (13).

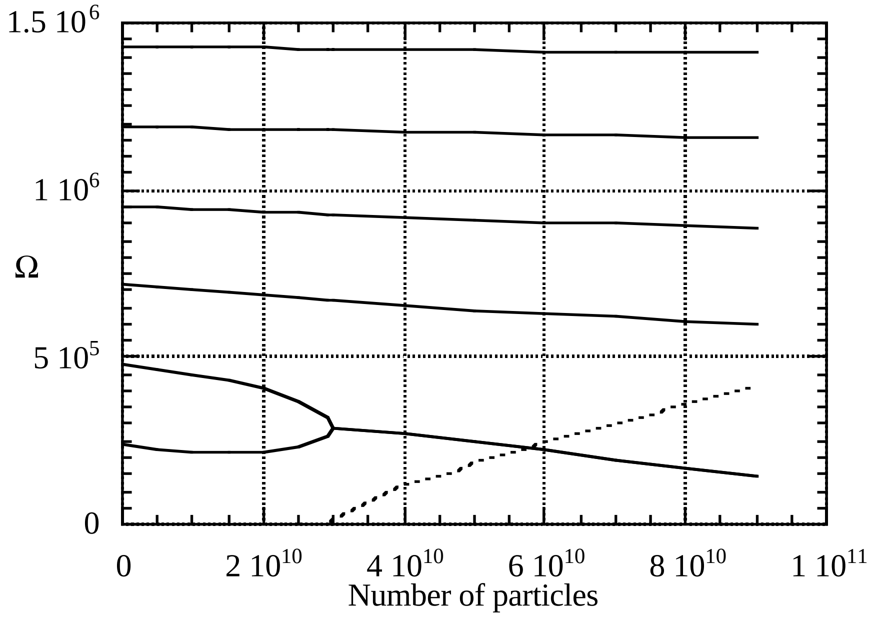

FIGURE 8. Azimuthal coherent frequency shifts.

In the phase space it is described by the equation:

$$\psi(J) = \overline{\psi}\left[(1-\Gamma)U(J_1 - J) + \Gamma U(J_2 - J)\right], \tag{12}$$

where the constant $\overline{\psi}$ is derived from the normalization condition, Γ is a parameter between 0 and 1 to better approximate the double water-bag to the real distribution, and $U(J)$ is the step function.

From equation (12), the radial modes are

$$R_m(J) = A_m \delta(J_1 - J) + B_m \delta(J_2 - J)\,, \tag{13}$$

with $\delta(J)$ the symbolic Dirac delta function. By using the relation (13) in equation (4), we obtain the eigenvalue system:

$$\begin{cases} \left[\Omega - m\omega(J_1)\right] A_m = -(1-\Gamma) \displaystyle\sum_{l=-\infty}^{\infty} \left(M_{11}^{ml} A_l + M_{12}^{ml} B_l\right) \\ \left[\Omega - m\omega(J_2)\right] B_m = -\Gamma \displaystyle\sum_{l=-\infty}^{\infty} \left(M_{21}^{ml} A_l + M_{22}^{ml} B_l\right) \end{cases}, \tag{14}$$

with

$$M_{ij}^{ml} = i \frac{\alpha_c e^2 N}{8\pi^3 T_o E\left[(1-\Gamma)J_1 + \Gamma J_2\right]\omega(J_i)} \int_0^{2\pi} \varepsilon(J_i,\phi) e^{-im\phi} d\phi \int_0^{2\pi} e^{il\phi'} w\left[z(J_i,\phi) - z\left(J_j,\phi'\right)\right] d\phi' \,, \tag{15}$$

and where $w(z)$ is the machine wake field.

The determinant of the system (14) gives the values of the eigenfrequencies Ω. If we take into account also the coupling of the radial modes with different azimuthal numbers, we obtain for the accumulator ring the plot shown in Fig. 9. Even if the threshold, determined by the coupling of modes 4 and 5 with different radial number in this case is the same of that of the simulations $\left(N_{th} = 2\times10^{10}\right)$, it is necessary to point out that the truncation of the matrix to the first azimuthal modes (9 in this case) may lead to wrong results since the matrix elements go to infinity as m increases. The instability growth rate is lower than that obtained with the azimuthal mode coupling theory, but still higher than the radiation damping.

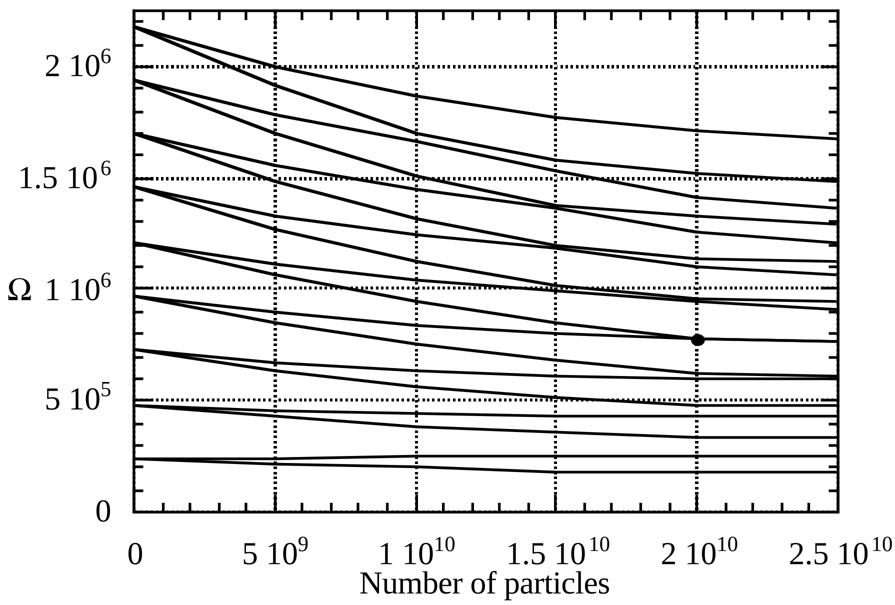

FIGURE 9. Azimuthal and radial coherent frequency shifts.

Conclusions

The numerical simulations show that the bunch in the DAΦNE accumulator ring is expected to be in the turbulent regime at the nominal current $N = 9 \times 10^{10}$. The microwave instability threshold has been found to be at $N_{th} = 2 \times 10^{10}$.

The bunch lengthens from the natural value of $\sigma_{zo} = 1.75$ cm to $\sigma_z = 4$ cm at the nominal current, and the relative energy spread widens from 4.1×10^{-4} to 6.5×10^{-4}.

Analysis of turn-by-turn beam sizes at the nominal current shows that the change in the bunch length does not exceed 4%, i. e. not dangerous for the machine operation.

Analytical study has been undertaken, and different models have been used to predict the microwave threshold value and compare it with the numerical results.

Application of Boussard criterion gives an instability threshold a factor 3 lower than that of the simulations.

The perturbation modal analysis of Vlasov equation gives a closer result. In particular it is shown that the bunch shape can be approximated by a gaussian distribution function, and the azimuthal mode coupling theory can be applied. The corresponding strong microwave instability threshold is found to be 1.5 times the one obtained with the simulations.

The double water-bag model allows to treat the radial mode coupling analytically. The threshold of a weak instability due to a coupling of radial modes with different azimuthal numbers has been found at $N_{th} = 2 \times 10^{10}$, which is exactly the same as found by numerical simulations.

Accurate experimental study is necessary to confirm the analytical predictions.

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