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DYNAMIC APERTURE WITH SYSTEMATIC AND RANDOM MULTIPOLE ERRORS IN DAΦNE

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1. INTRODUCTION

The presence of field errors in quadrupoles and bending magnets can reduce the beam stability area, called dynamic aperture. Hence a detailed study is needed, to set acceptable ranges for the multipolar component strengths and to find which are the more dangerous ones, in order to possibly modify the magnet design to minimize that specific component.

In this note we present the results obtained from simulations made for the DAΦNE main rings. The two structures[1] studied are the DAY-ONE lattice (without solenoids, normal conducting quadrupoles in the triplets) and the KLOE+KLOE lattice (D15, two equal Interaction Regions with high field solenoids and permanent magnet triplets).

In the first part the different sources of systematic and random field errors will be briefly discussed; in the following paragraph the results of the simulations performed on the two lattices will be presented and compared with the ideal dynamic apertures. The analysis has been performed separately for systematic and random components.

2. MULTIPOLE COMPONENTS IN A MAGNETIC FIELD

2.1 Systematic errors

First of all let us consider mechanically perfect magnets. The length and the actual shape of the magnet pole affect the ideal field distribution; the pole surface cannot be infinite but is laterally truncated to provide space for coils; moreover in the magnet design the ideal pole curvature is often approximated and not perfect. These are the main sources of the systematic multipole components, equal for all the magnets of the same type (same design and construction method). These higher order components have the same orientation and symmetry of the main one.

The number of higher order multipoles is minimized when the magnet is designed with the symmetry of the main component. However, we can still have a large number of systematic multipoles if a mechanical asymmetry shows up in the same way for all the magnets of the same kind.

As an example let us consider a symmetric quadrupole magnet: the pole finite width induces a 12-pole field, while the approximation to its ideal hyperbolic pole contour with a different shape, like a circular one, brings in the 20-pole component, whose intensity depends on how much the actual shape deviates from the ideal one.

In our simulations the systematic multipole contributions have been obtained from the field expansions delivered by magnet design programs.

2.2 Random errors

Random multipoles depend on magnet assembly and mechanical construction tolerances and are therefore different for each magnet. Pole flatness, parallelism and roughness can generate random errors, as well as the lack of a symmetric assembly[2]. In this case one assumes that in the field expansion the higher harmonic components follow statistical distributions over the total amount of magnets in the machine; the standard deviations of these error distributions come from measured field data or from construction specifications.

For their nature, the random multipole effect on beam dynamics can be significantly estimated only by performing the simulation with several error distributions, i.e. by finding out the dynamic apertures of a large number of randomly extracted sets of multipole coefficients.

3. DYNAMIC APERTURE STUDY

In order to study the dynamic aperture sensitivity to both random and systematic multipole errors we have used the well known code PATRICIA[3] and a home-developed code, TRACKMULT[4], since PATRICIA does not foresee the solenoidal fields needed for the DAΦNE detectors. In both programs the higher harmonic contributions to the ideal field are treated as thin lens kicks to the particle trajectory. The magnetic field in the horizontal mid-plane can be written as a power expansion:

$$B = B_0\rho_0 \sum_{n=0}^{m} \frac{1}{n!} k_n x^n \qquad (1)$$

where *n* indicates the 2(n+1) multipolar term, the k_n *[$m^{-(n+1)}$]* coefficients are the strengths of the multipole components and *x* *[m]* is the horizontal particle position with respect to the central trajectory. The field B is in *[Tesla]*.

From eq. (1) the normalized strength for each multipole component can be derived:

$$\frac{k_n}{k_i} = \frac{\Delta B_n}{B_i} \frac{n!}{x^{n-i}} \qquad [m^{-(n-i)}] \qquad (2)$$

and the error field at a certain distance x is:

$$\frac{\Delta B_n}{B_i} = \frac{k_n}{k_i} \frac{x^{n-i}}{n!}$$

where B_i is the ideal (linear) magnetic field in dipoles and quadrupoles, k_i is the strength coefficient relative to B_i (fundamental term of the expansion) and ΔB_n is the field contribution of the multipole of order n.

In order to avoid possible misunderstandings we also report the field expansion formula used in the magnet design codes (Magnet, Poisson):

$$B(x) = \sum_{n=1}^{m} M_n x^{n-1} \qquad (3)$$

where B is in *[Gauss]*, the coefficients M_n are expressed in *[Gauss/cm^{n-1}]* and x is in *[cm]*. Finally, from (2) and (3) the relation between multipolar coefficients k_n and M_n is easily obtained:

$$\frac{M_{n+1}}{M_{i+1}} = \frac{k_n}{k_i} \frac{10^{-2(n-i)}}{n!} \qquad [m^{-(n-i)}]$$

3.1 Systematic errors simulation

Systematic multipole errors were inserted in bendings, quadrupoles and in the low-β triplets, since their influence in the sextupoles has been found out to be negligible.

Sextupole, octupole and decapole components have been inserted in bending magnets, while 12-poles and 20-poles have been considered in the quadrupoles.

The low-β triplet quadrupoles must be treated in a special way. In fact, because of the crossing angle at the I.P., particles pass off-axis in these quadrupoles. This is a problem when tracking is performed with PATRICIA, which cannot take into account off-axis trajectories. The difficulty has been avoided by considering the low-β quadrupoles as gradient bending magnet for the simulations on the Day-one lattice. Of course, each multipole generates off-axis lower order harmonics, whose strengths depend on the multipole order and on the actual distance of the trajectory from the quadrupole axis. Therefore all the multipole components, from the 6-pole to the 20-pole, have been explicitly included in this simulation, taking as generating errors those belonging to the small quadrupoles (see Table I). The D15 lattice has been studied with TRACKMULT, which can deal with crossing angles, solenoidal fields and off-axis trajectories. Therefore only 12-pole and 20-pole components have been included into the input deck.

The bending magnets and some quadrupoles (the large aperture ones) will be built on the basis of LNF performed magnetic design. For this reason we extracted from the magnet designs and from the relative multipole field expansions the required set of normalized coefficients (k_n/k_i) to be used in our simulations. The *global* sensitivity to a given set of errors has been studied, comparing the ideal dynamic aperture (with sextupoles set to correct the natural machine chromaticity) to the one we get with multipole errors in all the lattice magnets at the same time.

The effect of a *single* multipole component for a selected magnet subset has also been studied, i.e. the dynamic aperture sensitivity has been investigated by putting only one systematic multipole component in all the magnets of the same type. This second approach has been followed in order to find which are the most dangerous multipole components for each type of magnet, giving therefore an estimate of the single multipole tolerance. It should be pointed out that, due to the non-linearity of the multipole fields, the combined effect of all the multipoles together in the magnetic lattice cannot be directly predicted from the contributions of different harmonics or magnet subsets.

For our *global* analysis, we have taken as systematic errors those estimated from the magnetic calculations performed during the storage ring design. Table I shows the normalized coefficients used for our simulations[5]. The corresponding values of the $\Delta B_n/B_i$ at a distance of *3 cm* from the magnet center are also indicated. The *large* quadrupoles are those in the wiggler arc, the *small* ones are all the others.

TABLE I - *Systematic multipole components*

(in parenthesis the corresponding $\Delta B_n/B$ @ 3 cm is shown)

ERROR LOCATION	$K_2/K_0[m^{-2}]$	$K_3/K_0[m^{-3}]$	$K_4/K_0[m^{-4}]$	$K_5/K_1[m^{-4}]$	$K_9/K_1[m^{-8}]$
BENDS	-8.4×10^{-2}	1.33	-97.13	-	-
	(-3.8×10^{-5})	*($6. \times 10^{-6}$)*	*(-3.3×10^{-6})*	-	-
SMALL	-	-	-	-4.21×10^{4}	1.94×10^{13}
QUADS	-	-	-	*(-2.8×10^{-4})*	*(3.5×10^{-5})*
LARGE	-	-	-	3.36×10^{2}	1.29×10^{14}
QUADS	-	-	-	*(2.4×10^{-6})*	*(2.4×10^{-4})*

In order to compare the results for the different multipole configurations we define as *required aperture* [6] a rectangle in the phase plane between **($-10\sigma_x$, $+10\sigma_x$)** off coupling in the horizontal plane and **$10\sigma_y$**, full coupling, in the vertical one (the vertical dynamic aperture is symmetric with respect to the horizontal axis, when there are no skew elements). This aperture allows for a Touschek beam lifetime of about 4 hours.

In the following the results of our simulations are summarized by listing the maximum stable horizontal amplitude, in σ_x units, with **at least 2 σ_y** stable vertical amplitude, inside the required aperture (in many cases the dynamic aperture is larger than the required $10\sigma_x$ value!). For all the simulations the Touschek and total beam lifetimes are also given, as computed in Ref.[7]. These values have to be compared with the values obtained for an ideal dynamic aperture of $10\sigma_x$ up to 1.5% energy deviation: **τ_{Tou} = 301 min** and **τ_{tot} = 220 min**. All the dynamic apertures are computed at the IP, with 256 turns and for three momentum deviations: 0,-1%,+1% (fixed energy).

Table II shows the results obtained from the *global* analysis for the Day-one lattice. As expected, the largest contribution to dynamic aperture reduction comes from the low-β quadrupoles, for particles with an energy deviation of +1%, corresponding to a reduction of about 4% on the total beam lifetime.

TABLE II

Day-one lattice results: apertures with systematic errors.
Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude

ERROR LOCATION	Δp/p = -1%	Δp/p = 0	Δp/p = 1%	τ Tou (min)	τ tot (min)
Bendings	-10/+10	-10/+10	-7/+10	**287**	**212**
Quadrupoles (no low-β)	-10/+9	-10/+10	-7/+9	**287**	**212**
Quadrupoles (including low-β)	-10/+10	-10/+10	-6/+7	**283**	**210**
Bendings & quads (no low-β)	-10/+9	-10/+10	-7/+9	**287**	**212**
All magnets (including low-β)	-10/+10	-10/+10	-6/+7	**283**	**210**

Fig. 1 shows the dynamic apertures with the systematic errors of Table I in bendings and quadrupoles (low-β included), compared to the ideal one. The stronger reduction occurs for +1% energy deviation, which presents a smaller dynamic aperture also without multipole errors.

The effect of a *single* multipole component is shown in Tables III.a and III.b as a function of $\Delta B_n/B_i$ for different energy deviations, for quadrupoles and bendings respectively. The tolerance on single multipole component contributions can be estimated again from their effect on the beam lifetimes. For the arc quadrupoles, both the 12-pole and 20-pole errors can be tolerated up to a value of $\pm 5\text{x}10^{-4}$. The reduction of the stable region, due to errors in the low-β triplets, is strong: a 12-pole component of about $-5\text{x}10^{-4}$ is still acceptable, while the dynamic aperture seems to be strongly affected by a 20-pole component ranging from $-5\text{x}10^{-4}$ to $\pm 10^{-4}$. It has to be pointed out that the considered values correspond to a very strong error on the pole profile design, that is unlikely to occur in practice.

For what concerns the bending magnets, a 6-pole and a 10-pole component can be safely tolerated up to a value of $7.5 \text{ x } 10^{-4}$ and $5 \text{ x } 10^{-4}$ respectively; for the 8-pole harmonic a value of $-5 \text{ x } 10^{-4}$ is acceptable, while $\Delta B/B = 5 \text{ x } 10^{-4}$ gives a total lifetime reduction to about 2 hours.

Comparing these limits with the $\Delta B_n/B_i$ values shown in Table I, it is clear that the LNF designed bendings have multipolar components well below the tolerable limits, while a particular care must be put in constructing the low-β quadrupoles.

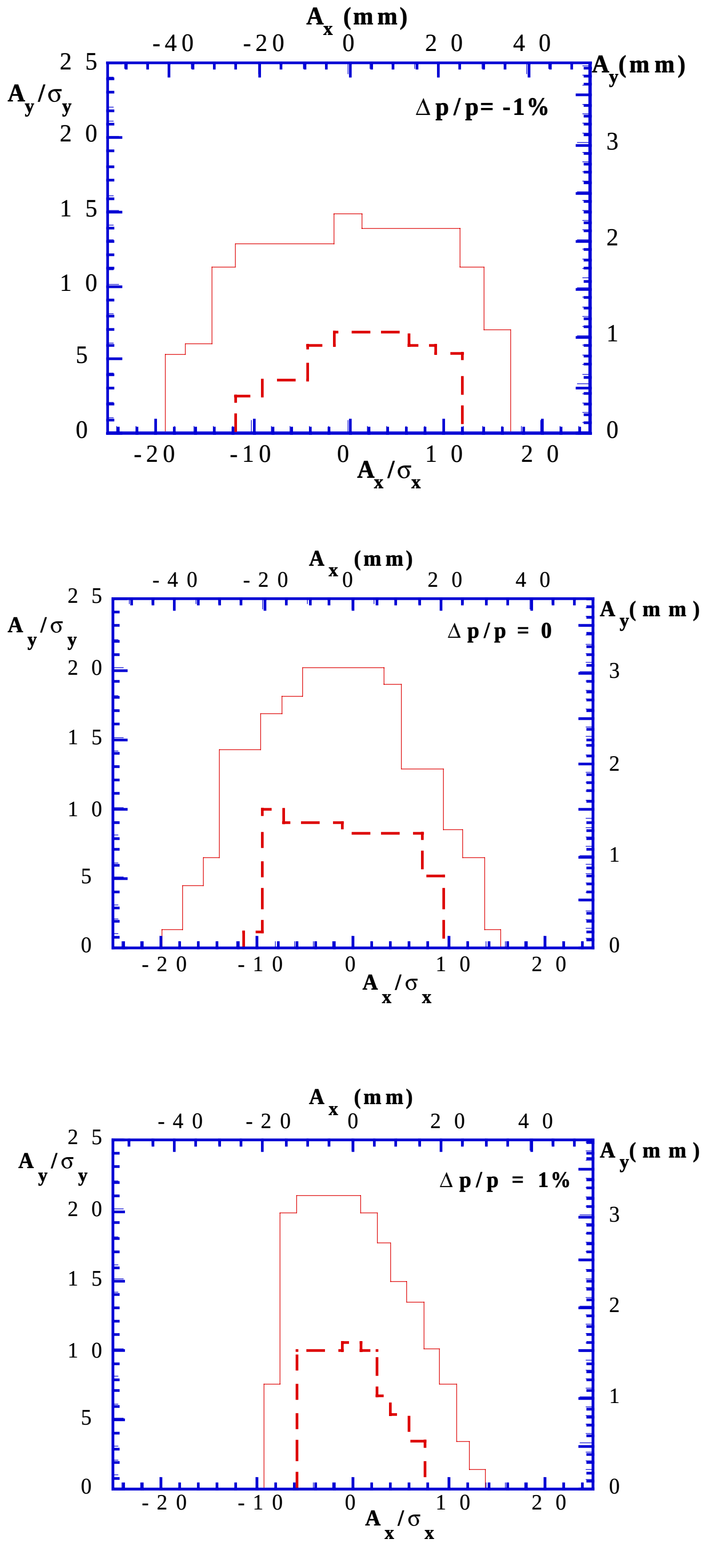

Fig. 1 - *Day-one Lattice: effect of systematic multipoles. Ideal dynamic aperture (solid line) compared to the one with all the systematic errors (dashed line) for three energy deviations.*

TABLE III.a

Day-one lattice results: analysis of dynamic aperture sensitivity to single systematic errors in quadrupoles. Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude

12-POLE IN QUADS (low-β quads without errors)

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-5x10^{-4}$	$5x10^{-4}$	$-7.5x10^{-4}$	$7.5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+10	-10/+9	-10/+9	-9/+6	-9/+6
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10
$\Delta p/p$ =+1%	-9/+10	-7/+9	-7/+10	-7/+10	-7/+10	-6/+9
τ_{Tou} (min)	**293**	**287**	**287**	**287**	**283**	**283**
τ_{tot} (min)	**216**	**212**	**212**	**212**	**210**	**210**

20-POLE IN QUADS (low-β quads without errors)

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-2.5x10^{-4}$	$2.5x10^{-4}$	$-3x10^{-4}$	$3x10^{-4}$	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-9/+9	-9/+9	-9/+6	-9/+6	-9/+6	-9/+6	-6/+6	-6/+6
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10
$\Delta p/p$ = 1%	-7/+10	-7/+9	-7/+9	-7/+9	-6/+9	-7/+7	-7/+9	-6/+7
τ_{Tou} (min)	**287**	**287**	**283**	**283**	**283**	**283**	**283**	**283**
τ_{tot} (min)	**212**	**212**	**210**	**210**	**210**	**210**	**210**	**210**

12-POLE IN LOW-β QUADS ONLY

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-6/+6	-10/+9	-9/+9	-4/+4
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-7/+7
$\Delta p/p$ = 1%	-7/+10	-7/+10	-6/+6	-3/+4
τ_{Tou} (min)	**283**	**287**	**283**	**140**
τ_{tot} (min)	**210**	**212**	**210**	**120**

20-POLE IN LOW-β QUADS ONLY

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-3x10^{-4}$	$3x10^{-4}$	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-4/+4	-6/+4	-4/+4	-4/+4	-4/+4	-4/+4
$\Delta p/p$ = 0	-10/+7	-10/+7	-7/+7	-7/+7	-7/+7	-7/+7
$\Delta p/p$ = 1%	-6/+7	-6/+6	-6/+4	-6/+6	-6/+6	-3/+4
τ_{Tou} (min)	**149**	**149**	**149**	**149**	**149**	**140**
τ_{tot} (min)	**126**	**126**	**126**	**126**	**126**	**120**

TABLE III.b

Day-one lattice results: analysis of dynamic aperture sensitivity to single systematic errors in bendings. Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude

6-POLE IN BENDS

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-5x10^{-4}$	$5x10^{-4}$	$-7.5x10^{-4}$	$7.5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+9	-10/+10	-6/+9	-10/+10	-6/+6
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10
$\Delta p/p$ = 1%	-7/+10	-7/+10	-7/+9	-6/+7	-7/+7	-6/+7
τ_{Tou} (min)	**287**	**287**	**287**	**283**	**287**	**283**
τ_{tot} (min)	**212**	**212**	**212**	**210**	**212**	**210**

8-POLE IN BENDS

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-2.5x10^{-4}$	$2.5x10^{-4}$	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+9	-10/+10	-6/+9	-10/+10	-6/+6
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-7/+7
$\Delta p/p$ = 1%	-7/+10	-7/+9	-7/+9	-6/+9	-6/+7	-4/+6
τ_{Tou} (min)	**287**	**287**	**287**	**283**	**283**	**149**
τ_{tot} (min)	**212**	**212**	**212**	**210**	**210**	**126**

10-POLE IN BENDS

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-2.5x10^{-4}$	$2.5x10^{-4}$	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+9	-9/+6	-9/+6	-4/+6	-9/+6
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10
$\Delta p/p$ = 1%	-9/+10	-7/+9	-9/+10	-7/+9	-6/+7	-7/+9
τ_{Tou} (min)	**293**	**287**	**283**	**283**	**260**	**283**
τ_{tot} (min)	**216**	**212**	**210**	**210**	**197**	**210**

A similar study has been carried out for the D15 lattice, which exhibits a larger ideal dynamic aperture, assuming the **same systematic errors** as for the Day-one. The results, summarized in Table IV, V.a and V.b., show that this lattice is less sensitive to multipole errors than the previous one. The dynamic aperture with all errors included (Table IV) is plotted in Fig. 2 as compared to the ideal one.

TABLE IV

D15 lattice results: apertures with systematic errors (from computer codes). Number of stable σ_x (off c.) for $2\sigma_y$ (full c.) stable vertical amplitude

ERROR LOCATION	Δp/p = -1%	Δp/p = 0	Δp/p = 1%	τ_{Tou} (min)	τ_{tot} (min)
Bendings	-10/+10	-10/+10	-10/+10	**301**	**220**
Quadrupoles (no low-β)	-10/+10	-10/+10	-8/+8	**290**	**214**
Quadrupoles (including low-β)	-10/+10	-10/+10	-10/+8	**290**	**214**
Bendings & quads (no low-β)	-10/+10	-10/+10	-8/+8	**290**	**214**
All magnets (including low-β)	-10/+10	-10/+10	-10/+8	**290**	**214**

A generalized tolerance of $5\text{x}10^{-4}$ is acceptable for all magnets, with the exception of the low-β ones. These quadrupoles will be of the permanent type, due to the small size required to ensure a wide acceptance to the detector. It is almost impossible to foresee the harmonic content of the field in these quadrupoles, since there is no symmetry in the design, and the field shape may also depend on the longitudinal coordinate along the magnet, so that it is unlikely that a specific high order component could be much larger than the others. The overall field quality will be $5\text{x}10^{-4}$ within a good field radius of 3 cm and a prototype is under construction. The higher order terms in the field integral will be measured by means of the rotating coil method and the measured values, for each magnet, will be included in the simulation.

TABLE V.a

D15 lattice results: analysis of dynamic aperture sensitivity to single systematic errors in quadrupoles. Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude.

12-POLE IN QUADS (low-β quads without errors)

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+10	-10/+10	-10/+10
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10
$\Delta p/p$ =+1%	-10/+10	-10/+10	-10/+10	-10/+10
τ_{Tou}(min)	**301**	**301**	**301**	**301**
τ_{tot} (min)	**220**	**220**	**220**	**220**

20-POLE IN QUADS (low-β quads without errors)

$\Delta B/B$ @ 3cm	-10^{-4}	10^{-4}	$-3x10^{-4}$	$3x10^{-4}$	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-5/+5
$\Delta p/p$ = 0	-10/+10	-10/+10	-10/+10	-10/+10	-10/+10	-9/+8
$\Delta p/p$ = 1%	-10/+10	-10/+8	-10/+9	-9/+8	-10/+8	-5/+5
τ_{Tou}(min)	**301**	**290**	**293**	**290**	**290**	**195**
τ_{tot} (min)	**220**	**214**	**216**	**214**	**214**	**157**

12-POLE IN LOW-β QUADS ONLY

$\Delta B/B$ @ 3cm	$-5x10^{-4}$	$5x10^{-4}$
$\Delta p/p$ =-1%	-10/+10	-10/+10
$\Delta p/p$ = 0	-10/+10	-10/+10
$\Delta p/p$ = 1%	-10/+10	-10/+8
τ_{Tou} (min)	**301**	**290**
τ_{tot} (min)	**220**	**214**

20-POLE IN LOW-β QUADS ONLY

$\Delta B/B$ @ 3cm	$-3x10^{-4}$	$3x10^{-4}$	$-5x10^{-4}$	$5x10^{4}$
$\Delta p/p$ =-1%	-7/+6	-8/+8	-5/+5	-8/+8
$\Delta p/p$ = 0	-9/+7	-10/+8	-8/+7	-9/+8
$\Delta p/p$ = 1%	-7/+7	-7/+7	-6/+6	-7/+7
τ_{Tou} (min)	**157**	**201**	**195**	**201**
τ_{tot} (min)	**132**	**161**	**157**	**161**

TABLE V.b

D15 lattice results: analysis of dynamic aperture sensitivity to single systematic errors in bendings. Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude.

6-POLE IN BENDS

ΔB/B @ 3cm	-10^{-4}	10^{-4}	-5×10^{-4}	5×10^{-4}
Δp/p =-1%	-10/+10	-10/+10	-10/+10	-10/+10
Δp/p = 0	-10/+10	-10/+10	-10/+10	-10/+10
Δp/p = 1%	-10/+10	-10/+10	-10/+10	-10/+10
τ_{Tou} (min)	**301**	**301**	**301**	**301**
τ_{tot} (min)	**220**	**220**	**220**	**220**

8-POLE IN BENDS

ΔB/B @ 3cm	-10^{-4}	10^{-4}	-5×10^{-4}	5×10^{-4}
Δp/p =-1%	-10/+10	-10/+10	-10/+10	-9/+8
Δp/p = 0	-10/+10	-10/+10	-10/+10	-8/+7
Δp/p = 1%	-10/+10	-10/+10	-9/+7	-7/+7
τ_{Tou} (min)	**301**	**301**	**287**	**157**
τ_{tot} (min)	**220**	**220**	**212**	**132**

10-POLE IN BENDS

ΔB/B @ 3cm	-10^{-4}	10^{-4}	-5×10^{-4}	5×10^{-4}
Δp/p =-1%	-10/+10	-10/+10	-10/+10	-10/+10
Δp/p = 0	-10/+10	-10/+10	-10/+10	-10/+10
Δp/p = 1%	-10/+10	-10/+10	-8/+8	-10/+10
τ_{Tou} (min)	**301**	**301**	**290**	**301**
τ_{tot} (min)	**220**	**220**	**214**	**220**

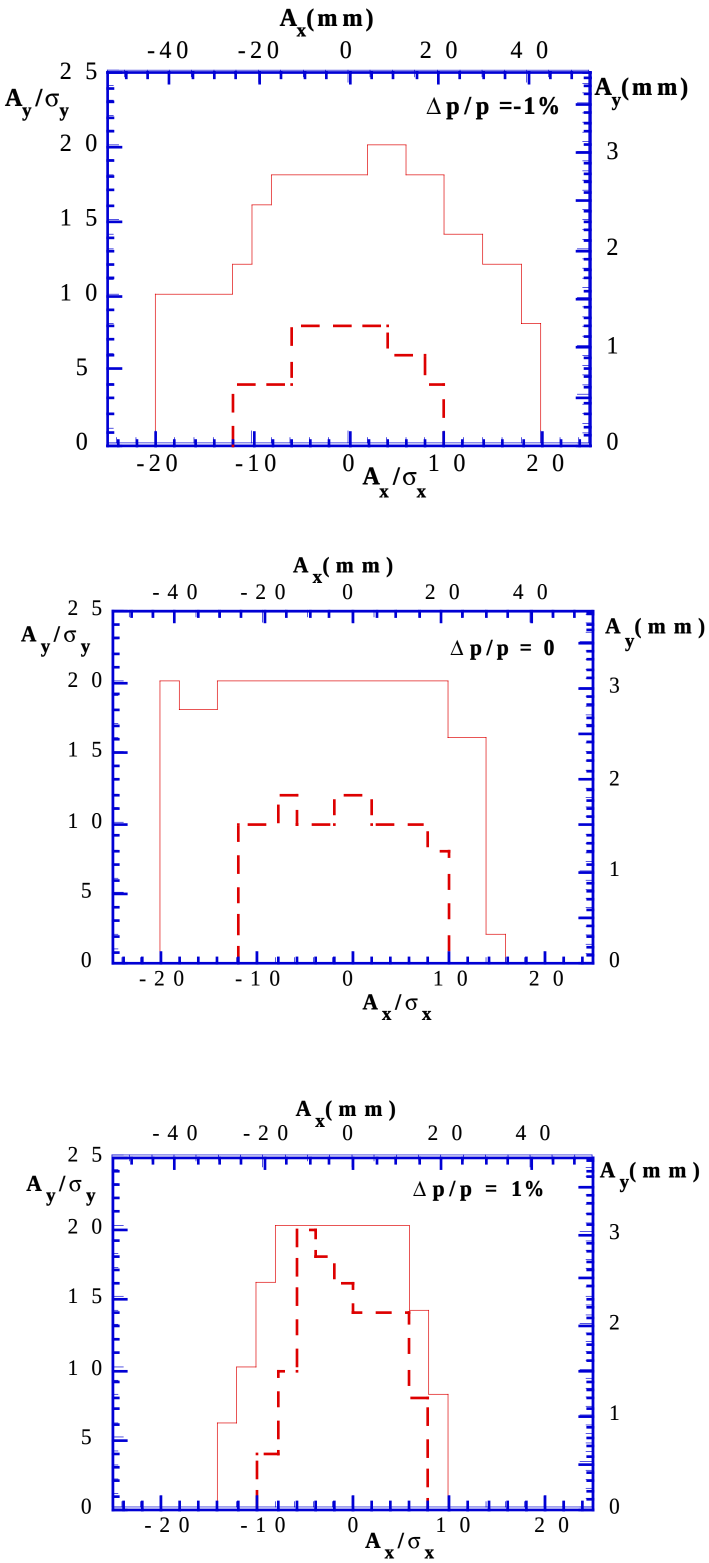

Fig. 2 - *D15 Lattice: effect of systematic multipoles. Ideal dynamic aperture (solid line) compared to the one with all the systematic errors (dashed line) for three energy deviations.*

In order to investigate a possible dependence of the machine sensitivity to systematic multipole errors on the chosen working point (Q_x=5.18, Q_y=6.15), a study of dynamic aperture as a function of the tunes has been performed. For the Day-one lattice it has been considered a 20-pole error in all the quadrupoles (see Table VI), for the D15 the dependence of the 20-pole error in the low-β triplets only has been also investigated (Tables VIII.a and VIII.b). For comparison in Table VII the dynamic aperture behaviour of D15 lattice as a function of the horizontal tune **without errors** is reported. The dependence on the tune for the ideal case is very weak, the largest reduction in the total beam lifetime being 3.6% for Q_x = 5.13, while the dynamic aperture in presence of 20-pole errors seems to be very sensitive to tune adjustments, since the reduction is important for each considered working point, ranging from 27% (Q_x = 5.18, chosen working point) to 61% (Q_x = 5.13). These results are summarized in Fig. 3, where the total beam lifetime is plotted as a function of the horizontal tune.

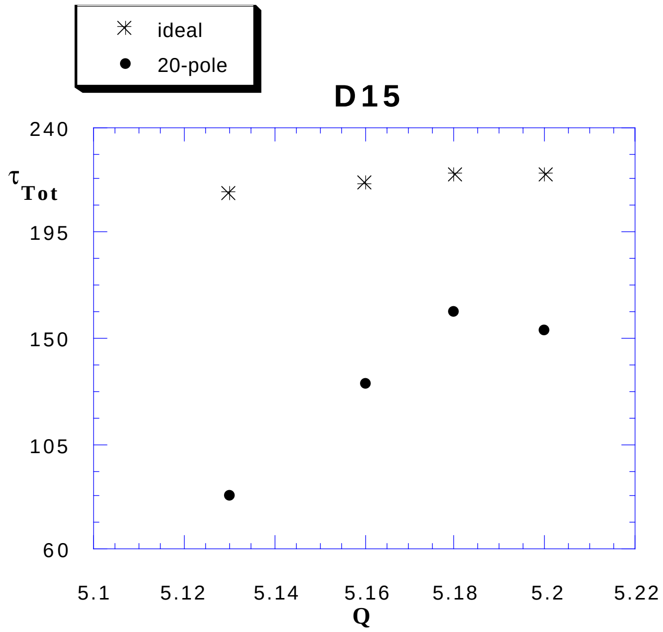

Fig. 3 - *D15 Lattice: dynamic aperture behaviour* vs. *horizontal tune for the ideal case (stars) and with 20-pole errors in low-β quads only (dots).*

TABLE VI

Day-one lattice results: analysis of dynamic aperture sensitivity to 20-pole systematic error in all quadrupoles as a function of the working point (Q_y = 6.15).
Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude.

$\Delta B/B = 5\times10^{-4}$	Q_x = 5.2	Q_x = 5.18	Q_x = 5.16	Q_x = 5.13
Δp/p =-1%	-8/+8	-4/+4	-4/+1	-1/+1
Δp/p = 0	-7/+6	-7/+7	-7/+7	-7/+6
Δp/p = 1%	-3/+3	-3/+4	-6/+6	-6/+5
τ_{Tou} (min)	**107**	**140**	**113**	**91**
τ_{tot} (min)	**81**	**120**	**100**	**71**

TABLE VII

D15 lattice results: analysis of dynamic aperture as a function of the working point (Q_y = 6.15). Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude.

$\Delta B/B = 5 \times 10^{-4}$	Q_x = 5.2	Q_x = 5.18	Q_x = 5.16	Q_x = 5.13
Δp/p =-1%	-10/10	-10/10	-10/10	-10/9
Δp/p = 0	-10/10	-10/10	-10/10	-10/10
Δp/p = 1%	-10/10	-10/10	-10/9	-10/7
$\tau_{\mathbf{Tou}}$ **(min)**	**301**	**301**	**293**	**287**
$\tau_{\mathbf{tot}}$ **(min)**	**220**	**220**	**216**	**212**

TABLE VIII.a

D15 lattice results: analysis of dynamic aperture sensitivity to 20-pole systematic error in low-β quadrupoles only as a function of the working point (Q_y = 6.15). Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude.

$\Delta B/B = 5 \times 10^{-4}$	Q_x = 5.2	Q_x = 5.18	Q_x = 5.16	Q_x = 5.13
Δp/p =-1%	-5/+5	-8/+8	-7/+6	-5/+5
Δp/p = 0	-9/+7	-9/+8	-9/+7	-9/+6
Δp/p = 1%	-4/+5	-7/+7	-8/+6	-7/+5
$\tau_{\mathbf{Tou}}$ **(min)**	**149**	**201**	**156**	**114**
$\tau_{\mathbf{tot}}$ **(min)**	**126**	**161**	**131**	**85**

TABLE VIII.b

*D15 lattice results: analysis of dynamic aperture sensitivity to 20-pole systematic error in **all** quadrupoles as a function of the working point (Q_y = 6.15). Number of stable σ_x (off c.) for 2 σ_y (full c.) stable vertical amplitude*

$\Delta B/B = 5 \times 10^{-4}$	Q_x = 5.2	Q_x = 5.18	Q_x = 5.16	Q_x = 5.13
Δp/p =-1%	-5/+5	-7/+7	-7/+7	-5/+4
Δp/p = 0	-9/+8	-9/+8	-9/+7	-9/+6
Δp/p = 1%	-4/+5	-7/+7	-8/+6	-7/+5
$\tau_{\mathbf{Tou}}$ **(min)**	**188**	**201**	**156**	**112**
$\tau_{\mathbf{tot}}$ **(min)**	**153**	**161**	**131**	**83**

3.2 Random errors simulation

In this case the multipole coefficients have been extracted from experimental measurements on existing magnets, performed at PEP, AGS, ALS[8]. The values used in our simulations[2] are conservative with respect to these data.

Table IX shows all the coefficients used as standard deviations of the Gaussian error distributions. For bendings and quadrupoles we included the 6-pole, 8-pole and 10-pole terms, adding for the quadrupoles also the 12-pole and 20-pole coefficients.

To represent the effect of random errors on the dynamic aperture we have chosen the same criterion used in the previous analysis: for each sequence of errors, corresponding to one machine, we take the value of the maximum stable horizontal amplitude, expressed in number of σ_x (off coupling) with at least $2\sigma_y$ (full coupling) stable vertical amplitude.

For what concerns the Day-one lattice, Fig. 4 shows the results obtained by simulating 50 machines without errors in the low-β quadrupoles, for three different particle energies. For particles on energy and for a deviation of -1%, the average values of these distributions, on both sides with respect to the axis origin, are above the limits of $\pm 10\ \sigma_x$, even if few machines exhibit a smaller dynamic aperture. For $\Delta p/p = +1\%$ the average value of the distributions is around $7\sigma_x$.

Including the errors also in the low-β quadrupoles, we obtain the results shown in Fig. 5. As one can see the resulting average dynamic aperture is still acceptable even if smaller than the previous one. Again for positive energy deviation the reduction is larger.

The vertical aperture behaviour, always well above the required limit of $10\ \sigma_y$, is not shown.

The same analysis has been performed on the D15 lattice for the worst case only, that is with random errors in all the quadrupoles, low-β included, for the same number of machines, tracking particles in the area corresponding to the required aperture ($-10\sigma_x$, $10\sigma_x$, $10\sigma_y$). The results are shown in Fig. 6, for three energy deviations. As for the systematic errors, this lattice is less sensitive to multipole random errors, even though there is a reduction for $\Delta p/p = +1\%$.

TABLE IX - *Random r.m.s. multipole components.*

(in parenthesis the correspondent $\Delta B_n/B$ @ 3 cm is shown)

	K_2/K_0 [m^{-2}]	K_3/K_0 [m^{-3}]	K_4/K_0 [m^{-4}]	K_5/K_1 [m^{-4}]	K_9/K_1 [m^{-8}]
BENDS	0.566 *(2.6×10^{-4})*	.85 *(3.8×10^{-6})*	567. *(1.9×10^{-5})*		
QUADS	0.015 *(2.3×10^{-4})*	2.4 *(3.6×10^{-4})*	60. *(6.8×10^{-5})*	3600. *(2.4×10^{-5})*	9.6×10^{10} *(1.7×10^{-7})*

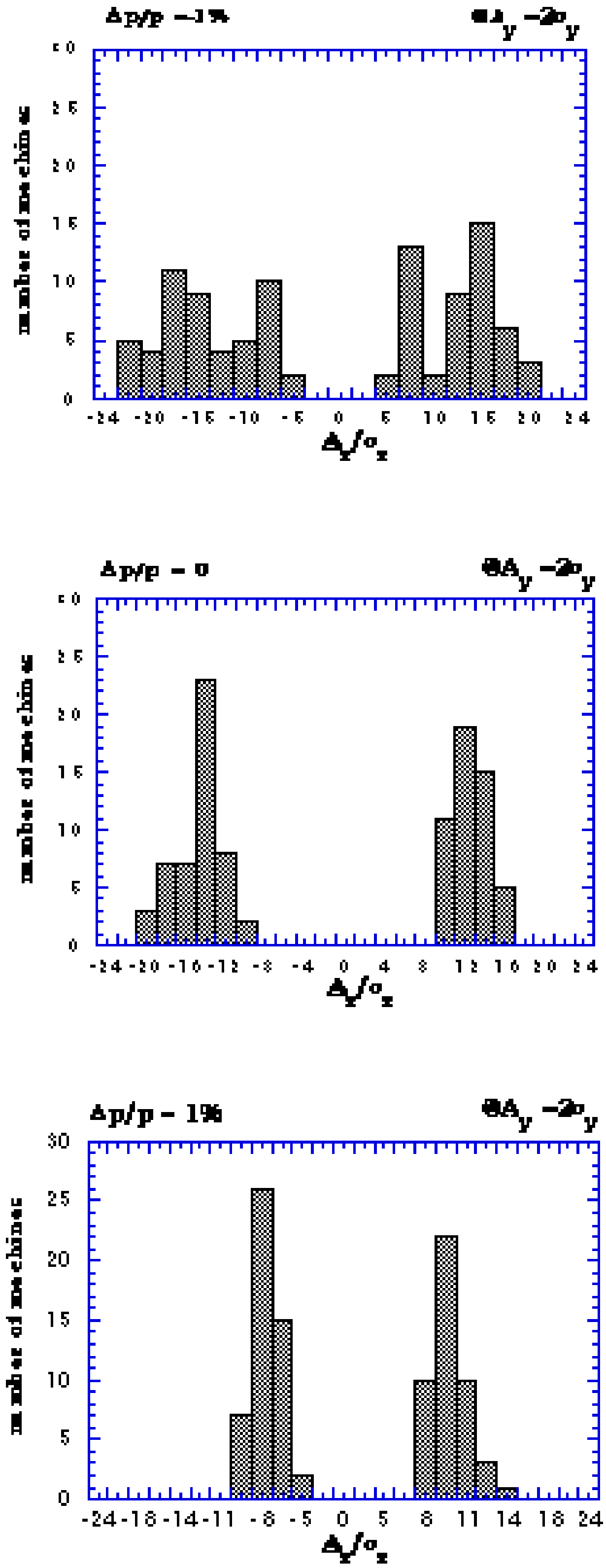

Fig. 4 - *DAY-ONE Lattice: effect of random multipole errors in all magnets except for low-β quadrupoles, on a sample of 50 machines, for three energy deviations.*

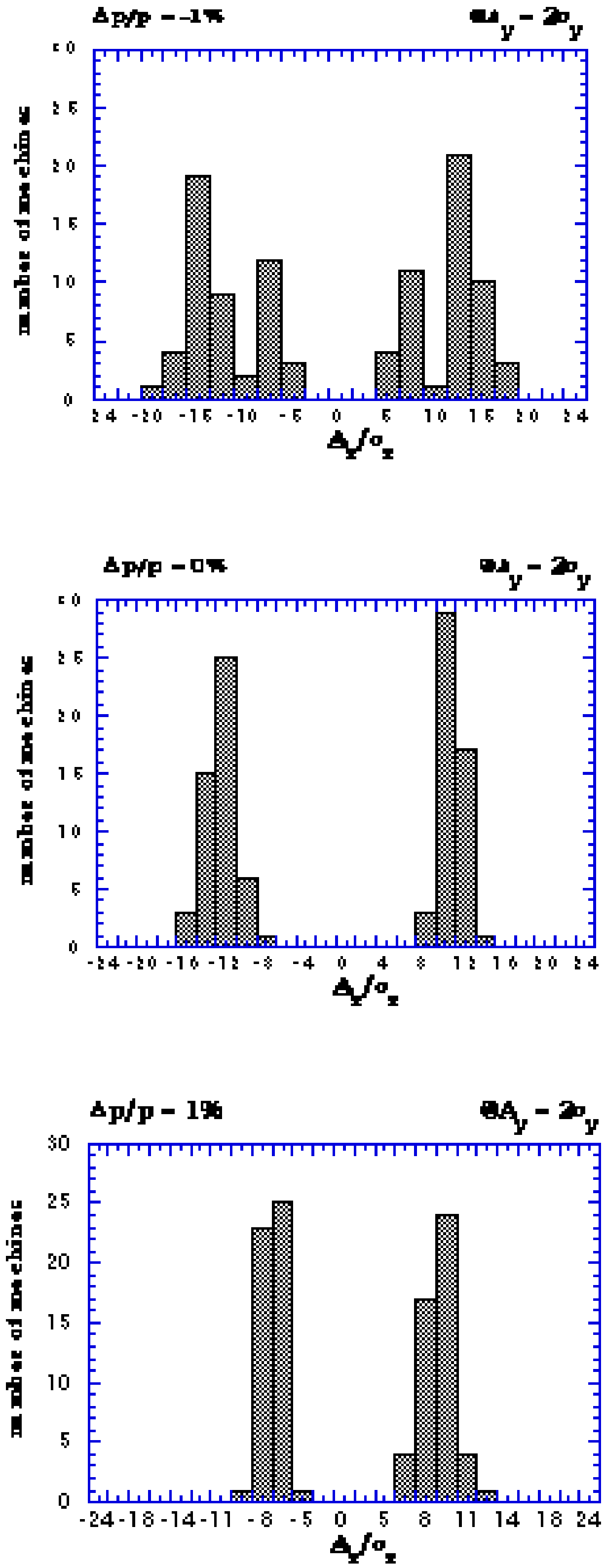

Fig. 5 - *Day-one Lattice: effect of random multipole errors in all magnets including the low-β quadrupoles, on a sample of 50 machines, for three energy deviations.*

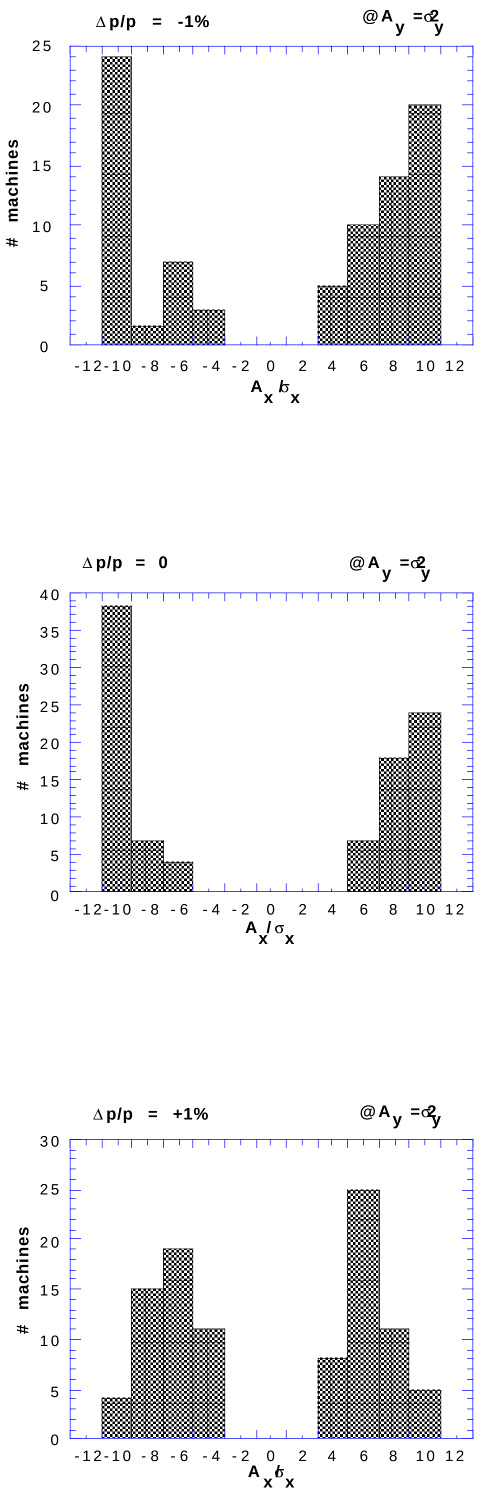

Fig. 6 - *D15 Lattice: effect of random multipole errors in all magnets including the low-β quadrupoles, on a sample of 50 machines, for three energy deviations.*

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