DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

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Note: **L-16**

DYNAMIC APERTURE SENSITIVITY TO MULTIPOLE ERRORS IN LOW-β PERMANENT MAGNETS

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The sensitivity of the dynamic aperture to the measured multipole errors has been studied to check the field quality of the two QF1 prototypes, the KLOE permanent magnets built by ASTER Ent.

The two prototypes, have an integrated gradient of 1.186 T each. They differ from each other only in the multipole distribution. The ASTER measurements of the multipole components at a radius R = 30 mm are listed in Table I for the two quadrupoles.

Table I - QF1 Multipole errors distribution

	Quad. #1		**Quad. #2**	
n	**B_n/B_1**	**Rel-phase(°)**	**B_n/B_1**	**Rel-phase(°)**
2	1.207×10^{-4}	-121.8	7.253×10^{-5}	102.9
3	1.407×10^{-4}	4.8	1.175×10^{-4}	60.6
4	1.151×10^{-4}	74.3	1.718×10^{-4}	-42.1
5	6.331×10^{-5}	168.7	9.671×10^{-5}	-86.0
6	7.288×10^{-5}	35.9	1.911×10^{-5}	-63.2
7	7.823×10^{-5}	-30.	5.735×10^{-5}	-133.2
8	2.192×10^{-5}	47.8	1.049×10^{-4}	-109.3
9	6.725×10^{-5}	-9.7	5.538×10^{-5}	167.2
10	6.950×10^{-6}	-1.3	1.754×10^{-5}	57.2
11	8.526×10^{-6}	-30.7	5.678×10^{-7}	-54.7
12	5.628×10^{-6}	11.6	3.120×10^{-6}	-110.7
13	1.365×10^{-5}	-4.9	1.065×10^{-5}	1.5
14	1.950×10^{-6}	89.7	1.957×10^{-6}	76.1
15	1.227×10^{-7}	-157.9	7.253×10^{-7}	161.8
16	3.489×10^{-7}	-110.5	5.651×10^{-7}	-108.9
17	5.093×10^{-6}	179.6	5.482×10^{-6}	179.5

A plot of the sum of B_x and B_y components of each multipole and their vector sum, calculated from B_n/B_1 and the relative (multipole vs. quadrupole) phase, is shown in Fig. 1.

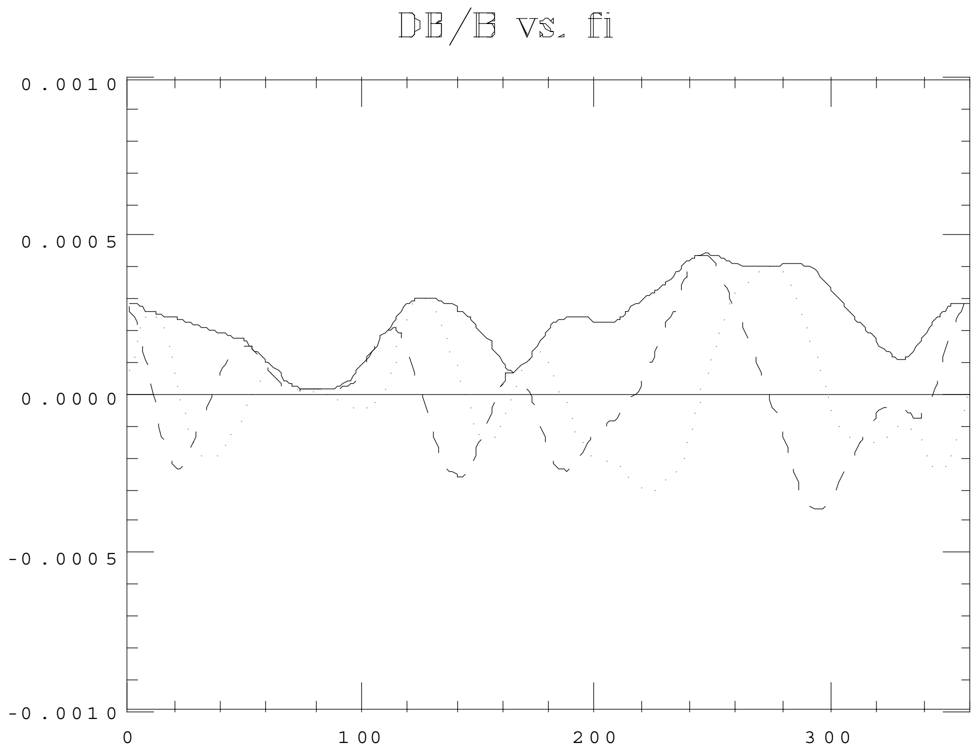


Fig. 1 - B_x (dotted line), B_y (dashed line), $|B|$ (solid line)

Each individual multipole component is below 1.7×10^{-4} and the average of $|B|$ (sum of all the components) is $\approx 2.5 \times 10^{-4}$, about half the specified limit (5×10^{-4}).

Let's now write the magnetic field harmonic expansion as:

$$B(z) = \sum_{n=0}^{m} C_n \; r^n \; e^{i(n\phi + \phi_n)}$$

where:

$n = 0$ for a dipole etc.,

C_n = multipole components = $\dfrac{1}{n!} \dfrac{d^n B}{dx^n}$

ϕ = observation phase,

ϕ_n = multipole field phase.

In the tracking program the kicks due to a multipole **n** are computed as:

$$\Delta x' = - S_n \, r^n \cos (n\phi - \phi_n - (n+1)\theta)$$

$$\Delta y' = S_n \, r^n \sin (n\phi - \phi_n - (n+1)\theta)$$

$$S_n = \frac{B_n}{B_1} \frac{K^2 L}{R^{(n-1)}} \qquad [m^{-n}]$$

where:

K^2L is the integrated quadrupole strength and ϕ and r are computed at each turn from the particle coordinates: $x = r \cos \phi$ and $y = r \sin \phi$;

θ is the quadrupole rotation angle, needed to compensate the KLOE field.

Only components from sextupole to twelve-pole (n=2,..,9) have been taken into account.

The results are shown in Figs. 3 and 4, where multipoles in only one quadrupole or both have been considered. In Fig. 2 the dynamic aperture without multipole errors is shown for comparison.

The results are satisfactory, showing that the measured multipoles do not deteriorate the dynamic aperture.

An example with all multipole components in the first quads set at 5×10^{-4} (very high value!) has been also run (Fig. 5), showing a reduction of the d.a. to 10 σ_x at $\Delta p/p = + .5$ %.

Fig. 6 shows results of the same simulation performed by applying the same error distributions of Table I, scaled by the quadrupole integrated strengths, to the two defocusing quadrupoles (QD) of the KLOE Interaction Region, placed in a high β_y position. A non negligible reduction of the vertical stable area can be noticed, especially for negative $\Delta p/p$; the limit is however still above 10 σ_y, with the vertical beam size calculated in full coupling.

We can conclude that the QF1 prototypes have a good field quality and will not negatively affect the dynamic aperture, for the QD's quadrupoles, which have a larger integrated gradient but are also longer and have a higher bore radius with respect to the QF1's, a particular care should be devoted in order to minimize the multipole field components.

A IR lattice with this QF1 integrated gradient value has been designed; the matching with the arcs is easy provided that:

- **the values of the QD and the QF2 integrated gradients are changed to -3.414 T and 1.324 T** respectively (with less than 5% variation);

- the splitter field is slightly modified (5‰) and its corrector (for the 12.5 mrad crossing angle) is set to .3 mrad.

KLOE+KLOE WITHOUT MULTIPOLE ERRORS

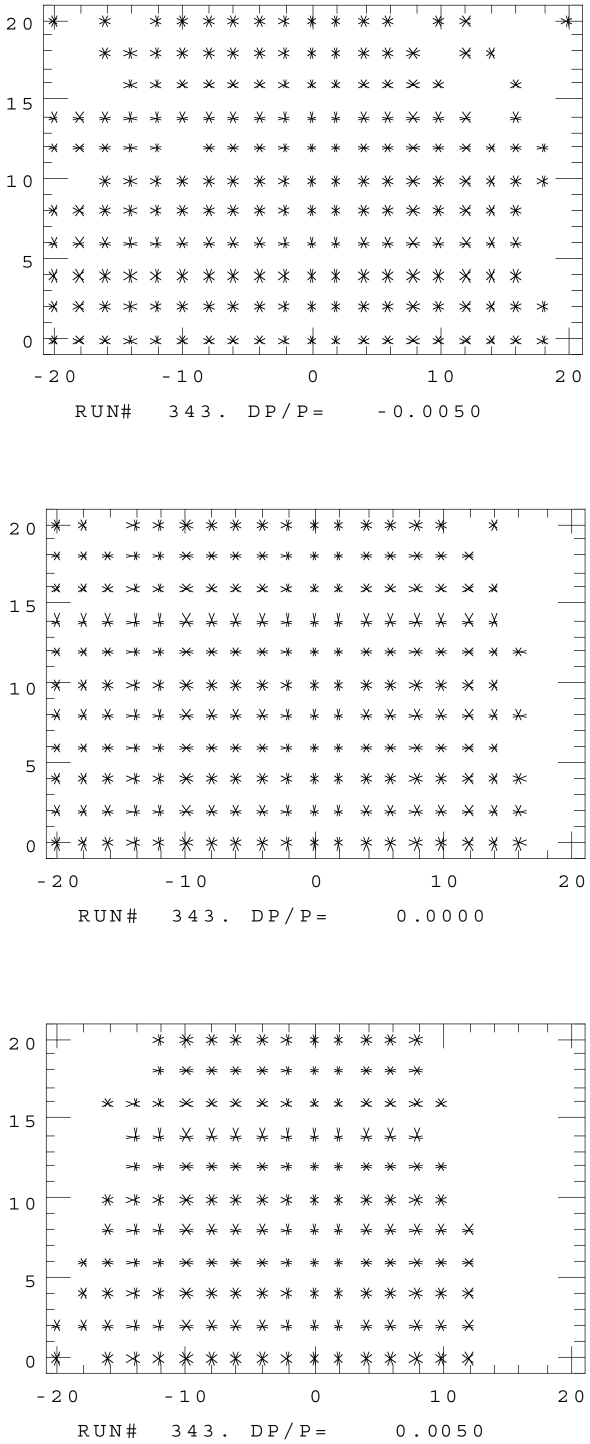
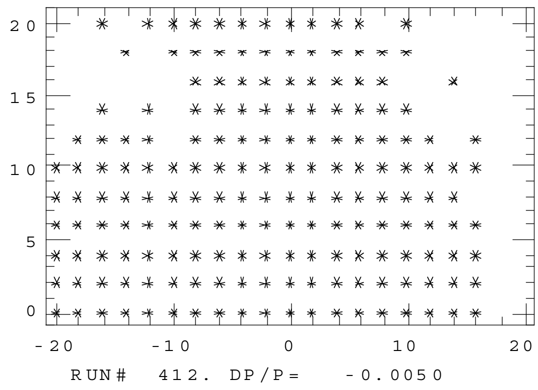
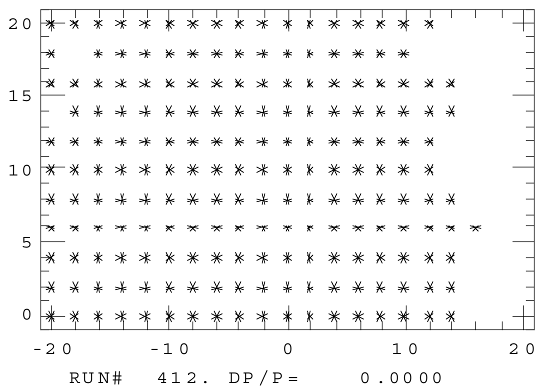


Fig. 2 - Ideal KLOE dynamic aperture.

KLOE+KLOE WITH MULTIPOLE ERR. IN QF1 ONLY





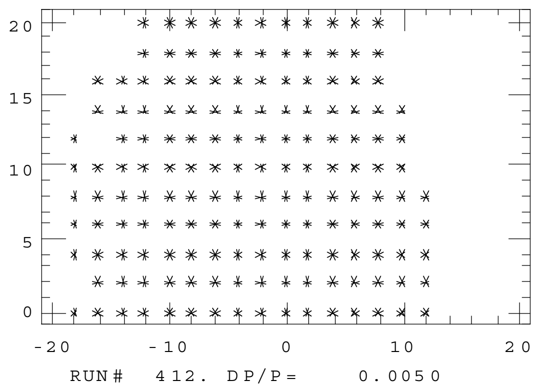


Fig. 3 - KLOE dynamic aperture with multipole errors of Table I in only one QF1.

KLOE+KLOE WITH MULTIPOLE ERR. IN QF1+QF1

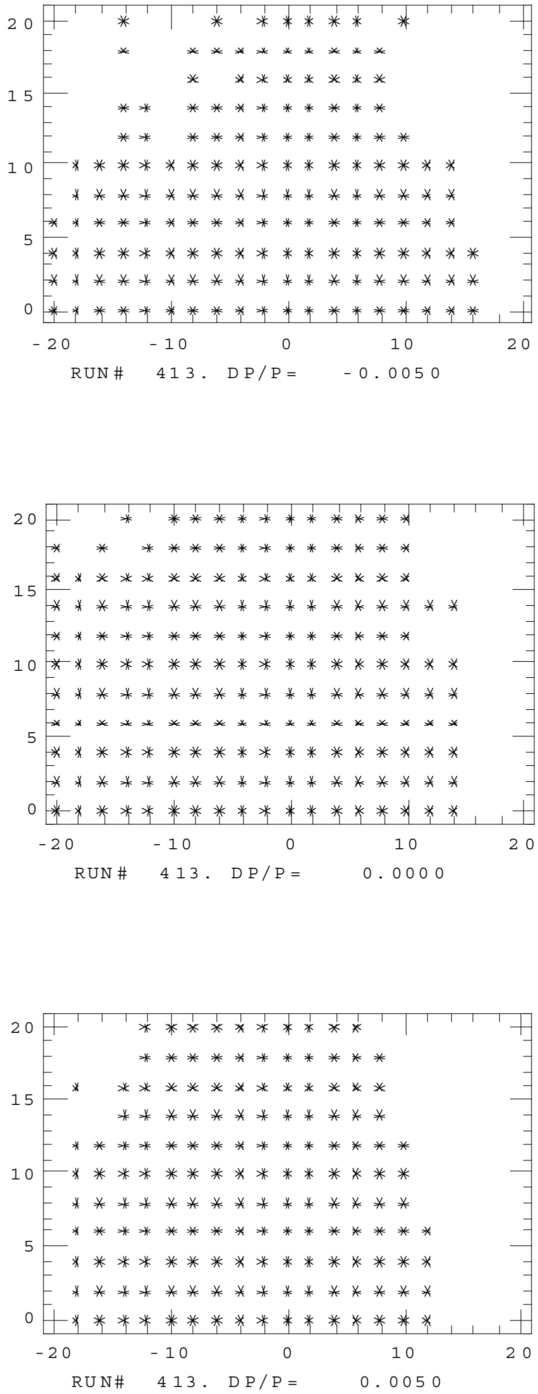
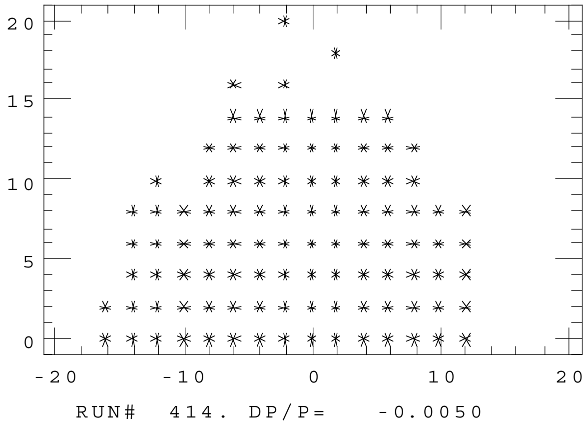
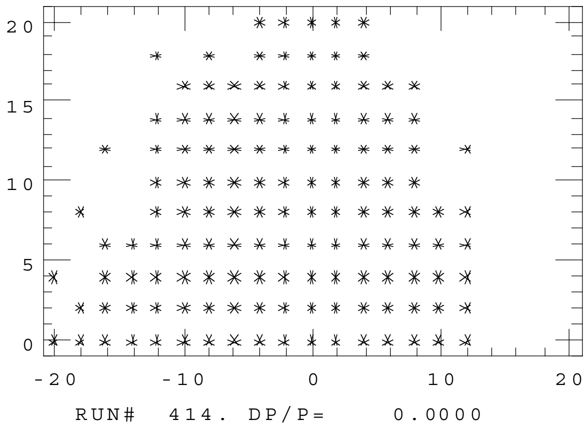


Fig. 4 - KLOE dynamic aperture with multipole errors of Table I in both QF1's .

WITH MULTIPOLE ERR. IN QF1+QF1 (DB/B = 5x10^{-4})





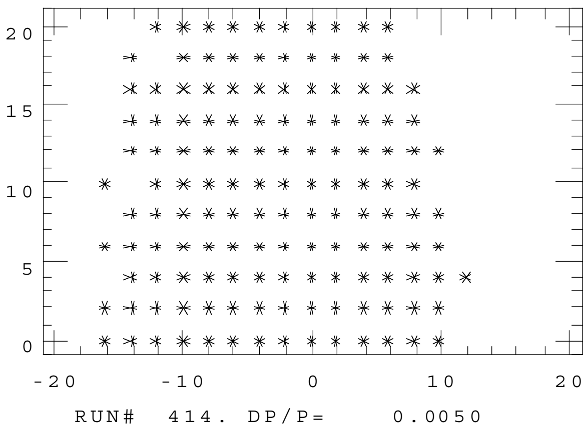


Fig. 5 - KLOE dynamic aperture with multipole errors of $B_n/B_1 = 5x10^{-4}$ *in both QF1's*

KLOE+KLOE WITH MULTIPOLE ERR. IN QD + QD

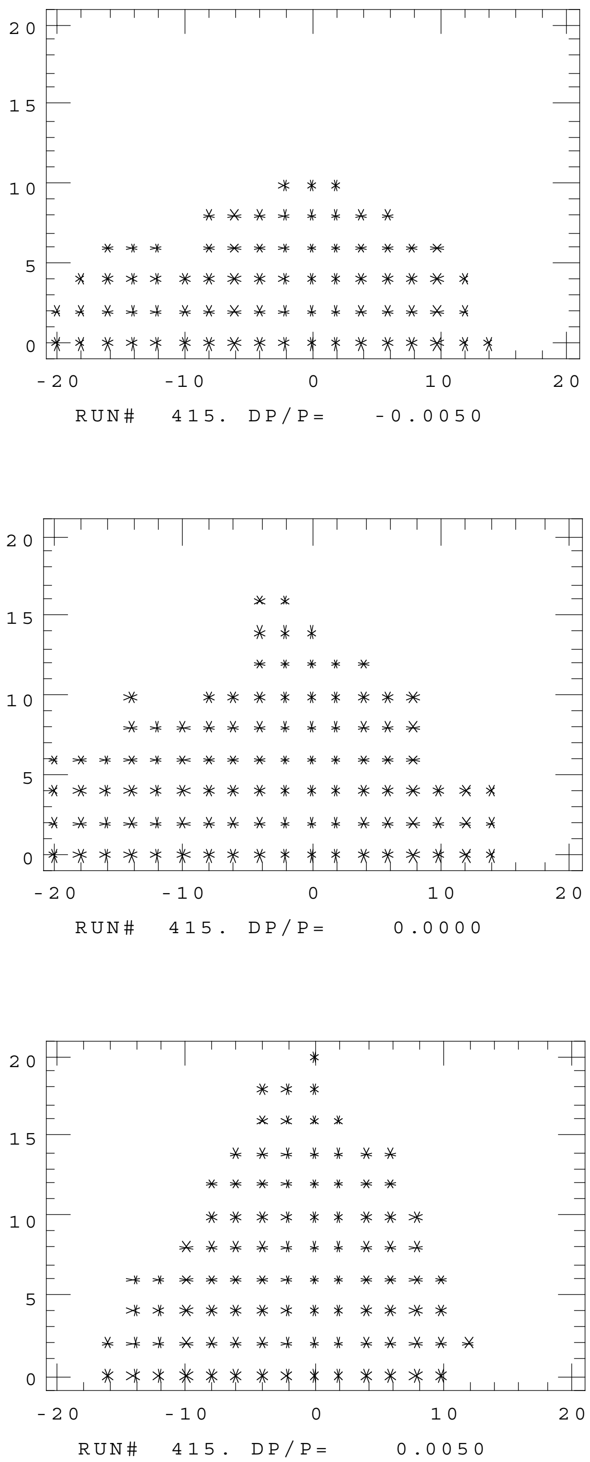


Fig. 6 - KLOE dynamic aperture with multipole errors of Table I in both QD's.