DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, March 1, 2001

Note: **M-5**

Design of an Octupole Magnet for DAΦNE

Claudio Sanelli[a], *Shiwen Han*[b]

[a]INFN, Laboratori Nazionali di Frascati, Frascati, Italy

[b] IHEP, Institute of High Energy Physics, Beijing,China

Since the trajectories of the electron and positron beams stored in the DAΦNE storage rings are off axis in the wiggler magnets, the decapole harmonic component of the magnetic field gives rise to an octupole component that affects the beam stability. To compensate this effect an octupole magnet has been designed. This note reports the design criteria and the results of the electromagnetic calculations.

1. Main parameters of the octupole magnet

1.1 Basic requirements:

1.1.1	Octupole constant :	$K = \frac{1}{B\rho} \times \frac{d^3B}{dX^3} \times \ell = 1000 \text{ m}^{-3}$
1.1.2	Magnetic length:	$\ell = 0.1$ m
1.1.3	Octupole bore radius:	$a = 0.05$ m
1.1.4	Beam energy :	E = 0.51 GeV
1.1.5	Bending dipole magnetic field:	B = 1.214 T

1.2 Calculation of the Ampere-turns/pole:

According to: $\frac{E}{0.3} = B\rho$

then: $B\rho = 1.7$ T.m

$$\frac{d^3B}{dX^3} = \frac{1000 \times B\rho}{\ell} = 1.7 \times 10^4 \text{ T/m}^3$$

$$T = \frac{1}{6}\frac{d^3B}{dX^3} = 2833.333 \text{ T/m}^3$$

$$NI = \frac{Ta^4}{4\mu_0} \times K_{iron} = 3875 \text{ A/pole}$$

where K_{iron} takes into account the saturation of the iron, $K_{iron} = 1.1$.

1.3 Choice of the coil conductor

1.3.1 We adopt a standard square hollow copper conductor:

Side of conductor	s=0.005 m
Diameter of the cooling hole	d=0.003 m
Radius of the rounded corners	r=1 mm

$$S = s^2 - \frac{\pi}{4}d^2 - (4r^2 - \pi r^2) = 17.07 \text{ mm}^2$$

1.3.2 Useful copper conductor area:

1.3.3 Number of turns per pole, N=33

Shape of the coil arrangement: See Fig. 1.

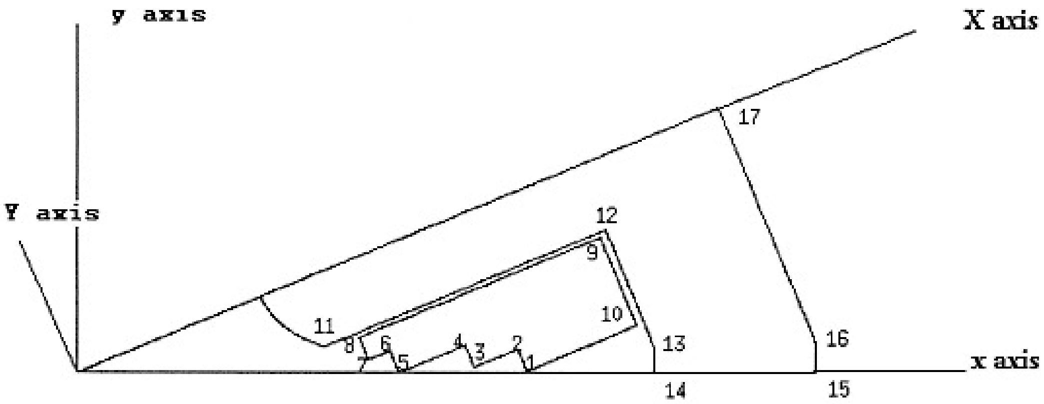

Figure 1: Sketch of the 1/16 octupole magnet.

1.4. Pole profile, coil and yoke:

1.4.1 Pole profile: the pole profile of an octupole must follow the equipotential line that satisfies the following equation in the X-Y coordinate system:

$$Y^4 + X^4 - 6X^2Y^2 = a^4$$

In the X-Y coordinate system the pole profile coordinates are listed in Table I.

Table I – Pole profile coordinates in the X-Y system

Points	X (cm)	Y (cm)
1	5.00	±0.000
2	5.05	±0.407
3	5.10	±0.575
4	5.15	±0.703
5	5.20	±0.811
6	5.25	±0.905
7	5.30	±0.989
8	5.40	±1.139
9	5.50	±1.270
10	5.60	±1.387
11	5.70	±1.495
12	5.80	±1.594
13	5.90	±1.687
14	6.00	±1.775

1.4.2 Coil

Considering an insulation thickness, half lapped, of turn insulation of 0.5 mm, the side of the isolated conductor turn is 6 mm. The coil coordinates, as shown in Fig. 1, are listed in Table II.

Table II – Coil coordinates in the X-Y system

Points	X (cm)	Y (cm)
1	10.5	±4.3
2	10.5	±3.7
3	9.3	±3.7
4	9.3	±3.1
5	7.5	±3.1
6	7.5	±2.5
7	6.9	±2.5
8	6.9	±1.9
9	13.5	±1.9
10	13.5	±4.3

1.4.3 Yoke: the coordinates of the iron yoke are listed in Table III.

Table III – Yoke coordinates in the X-Y system

Points	X (cm)	Y (cm)
11	6.000	±1.775
12	13.700	±1.775
13	13.700	±5.000
14	13.461	±5.576
15	17.195	±7.123
16	17.500	±6.387
17	17.500	±0.000

1.5. Coordinate conversion: we now change the coordinate system from the X-Y to the x-y one (x axis is on the median-plane) to simulate the symmetric octupole by means of the POISSON code, according to following coordinate transformation:

$$x = \sin\alpha \times Y + \cos\alpha \times X$$
$$y = \cos\alpha \times Y + \sin\alpha \times X$$

1.5.1 The pole profile coordinates are listed in Table IV.

Table IV – Coordinates of the pole profile in the x-y system

Points	x(cm)	y(cm)
1	4.619	1.913
2	4.821	1.557
3	4.932	1.420
4	5.027	1.321
5	5.115	1.241
6	5.200	1.173
7	5.275	1.115
8	5.425	1.014
9	5.567	0.931
10	5.705	0.862
11	5.838	0.800
12	5.968	0.747
13	6.096	0.699
14	6.223	0.656

1.5.2 The coil coordinate in the x-y system are listed in Table V.

Table V – Coil coordinates in the x-y system

Points	x(cm)	y(cm)
1	11.346	0.045
2	11.117	0.600
3	10.008	0.141
4	9.778	0.695
5	8.115	0.000
6	7.886	0.560
7	7.331	0.331
8	7.102	0.885
9	13.199	3.411
10	14.118	1.194

1.5.3 The yoke coordinates in the x-y system are listed in Table VI.

Table VI – Yoke coordinates in the x-y system

Points	x(cm)	y(cm)
11	6.223	0.656
12	13.336	3.603
13	14.570	0.623
14	14.570	0.000
15	18.612	0.000
16	18.612	0.796
17	16.168	6.697

1.6 Resistance, Voltage and Power

Since we have fixed the number of turns N=33, the needed current and current density will be:

$$I=NI/N=117.4\ A \qquad J=I/S=6.878\ A/mm^2$$

1.6.1 Coil conductor length: referring to Fig. 2, the number of turns in each sub-sections of the coil are as follows:

$$N1=20,\ N2=6,\ N3=6,\ N4=1;$$

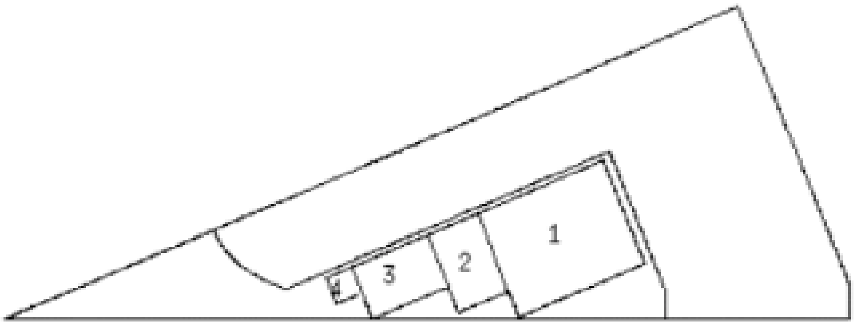

Figure 2: Sketch of the coil

For each section we calculate the conductor length, assuming L_{mech} = 6.5 cm:

Section 1 : $l_{1cp}=(2\pi R_{1cp}+2L_{mech})N_1=649.557$ cm ≈ 6.5 m
Section 2 : $l_{2cp}=(2\pi R_{2cp}+2L_{mech})N_2=183.558$ cm ≈ 1.9 m
Section 3 : $l_{3cp}=(2\pi R_{3cp}+2L_{mech})N_3=172.248$ cm ≈ 1.8 m
Section 4 : $l_{4cp}=(2\pi R_{4cp}+2L_{mech})N_4=26.823$ cm ≈ 0.27 m

the total length of the 8 coils will be:

$$l = 8\,(l_{1cp}+l_{2cp}+l_{3cp}+l_{4cp}) = 8\times 10.47 = 83.76 \text{ m}$$

1.6.2 Resistance

We assume a temperature of inlet water of 32^0 C and a inlet – outlet water temperature increase of $\Delta t = 20\ ^0$C. At 20 ^{0}C the coil resistance of the octupole will be:

$$R_{20} = \rho\times l/S = 84.4 \text{ m}\Omega$$

then

$$R_{42} = R_{20}\ (1+0.00433\times(42-20)) = 92.26 \text{ m}\Omega$$

Where the resistivity of copper at 20^0 is $\rho = 0.0172\ \Omega\cdot mm^2/m$ and the temperature coefficient of copper is 0.00433.

1.6.3 Voltage

$$V = IR = 10.8 \text{ V}$$

1.6.4 Power

$$P = I^2R = 1272 \text{ W}$$

1.7. Cooling water calculation

1.7.1 Assuming two water cooling circuits in parallel the water flow per circuit will be:

$$q = \frac{P(kW)}{4.186 \times \Delta t \times n} = 7.6 \times 10^{-3} \text{ litre/s} = 0.46 \text{ litre/min}$$

where n : number of parallel water circuit n = 2.

1.7.2 Water pressure calculation

1.7.2.1 Velocity of water

$$\upsilon = \frac{q \times 10^3}{\frac{\pi}{4} d^2} = 1.08 \text{ m/sec}$$

1.7.2.2 Pressure drop

$$\Delta P = 0.28 \frac{(l/n) \times \upsilon^{1.75}}{d^{1.25}} = 3.4 \text{ atm.}$$

2. Simulation

We calculate the magnetic field distribution of the octupole and its harmonic content by means of the POISSON code.

2.1. Results of the simulation

Because of the eight-fold symmetry of a symmetric octupole, we can limit the calculation to 1/16 of the octupole. The harmonic analysis is done interpolating 9 points over an arc of radius r_{int}=1.0 cm, with a normalization radius of 1.0 cm.

Figure 3 shows the magnetic flux lines which indicate the direction and strength of the magnetic field.

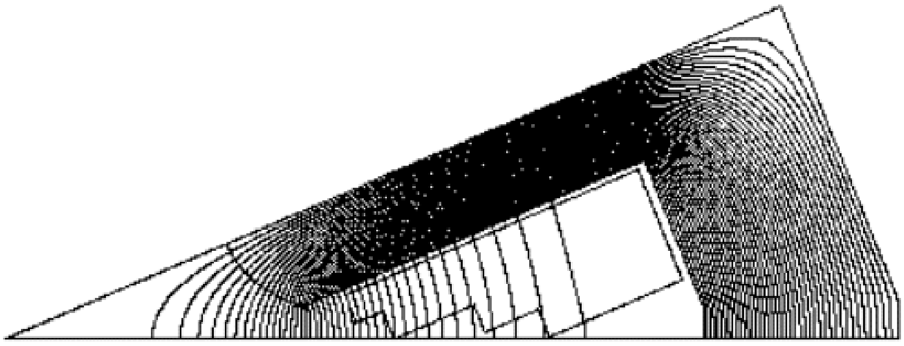

Figure 3: Magnetic flux lines of octupole magnet.

In the following we report the results obtained directly by the POISSON code.

```
 Interpolated points on an arc of radius =   1.00000
  K,L is nearest mesh point to physical coordinates X,Y:
          n     Angle       X          Y      K    L       Avector
          1    0.0000    1.0000    0.0000    51    1    7.52910E+00
          2    2.8000    0.9988    0.0488    51    4    7.38547E+00
          3    5.6000    0.9952    0.0976    51    7    6.96089E+00
          4    8.4000    0.9893    0.1461    50    9    6.27066E+00
          5   11.2000    0.9810    0.1942    51   12    5.34204E+00
          6   14.0000    0.9703    0.2419    50   15    4.20963E+00
          7   16.8000    0.9573    0.2890    49   17    2.91667E+00
          8   19.6000    0.9421    0.3355    49   20    1.51293E+00
          9   22.4000    0.9245    0.3811    47   23    5.12009E-02


  Vector potential coefficients
  Normalization radius =   1.00000
          A(x,y) = Re( sum (An + iBn) * (z/r)**n )
           n        An             Bn           Abs(Cn)
           4     7.5618E+00     0.0000E+00     7.5618E+00
          12     1.8502E-03     0.0000E+00     1.8502E-03
          20    -2.5177E-03     0.0000E+00     2.5177E-03
          28     3.4390E-03     0.0000E+00     3.4390E-03
          36    -4.4417E-03     0.0000E+00     4.4417E-03
          44     5.5953E-03     0.0000E+00     5.5953E-03


  Field coefficients
  Normalization radius =   1.00000
          (Bx - iBy) = i[sum n*(An + iBn)/r * (z/r)**(n-1)]
           n     n(An)/r        n(Bn)/r      Abs(n(Cn)/r)     Bn/B4
           4     3.0247E+01     0.0000E+00     3.0247E+01     1.0
          12     2.2202E-02     0.0000E+00     2.2202E-02     7.34E-04
          20    -5.0355E-02     0.0000E+00     5.0355E-02    -1.66E-03
          28     9.6292E-02     0.0000E+00     9.6292E-02     3.18E-03
          36    -1.5990E-01     0.0000E+00     1.5990E-01    -5.28E-03
          44     2.4619E-01     0.0000E+00     2.4619E-01     8.14E-03
```

2.2. Magnetic field distribution

2.2.1 Field distribution along x axis

As shown in Fig. 4, the magnetic field component By increases from the center of the magnet according to the third power of the abscissa till about 0.6 T. Then it goes down to zero in the coil region. Inside the iron yoke, the magnetic field reaches a peak greater than 0.8 T with an average value of about 0.75 T.

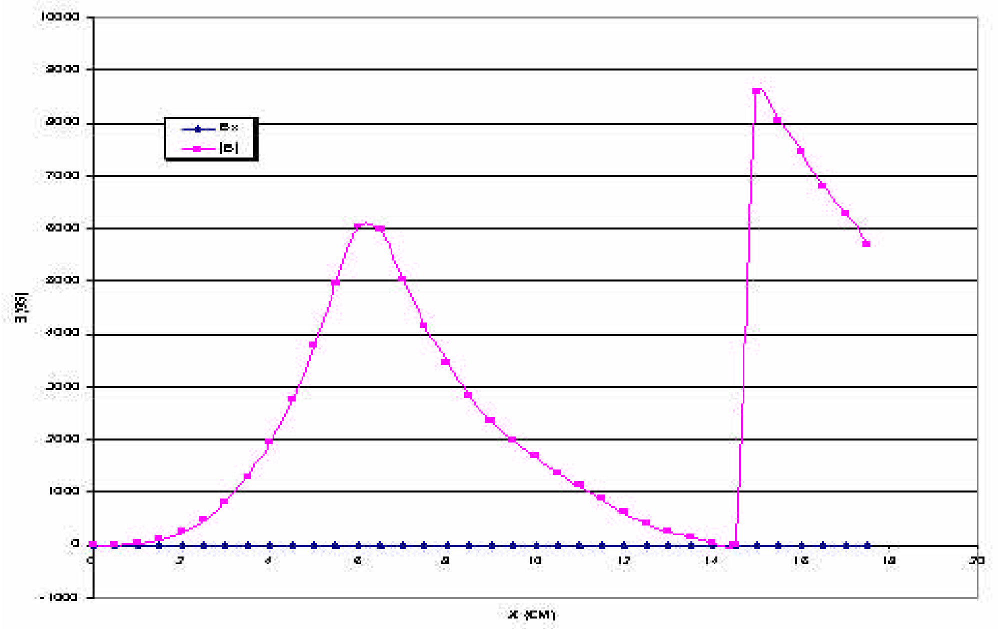

Figure 4: Magnetic field of the Octupole magnet along x axis

2.2.2. Field distribution in the bore region

In Fig. 5 the magnetic field distribution along a 22.4 degree line is shown.

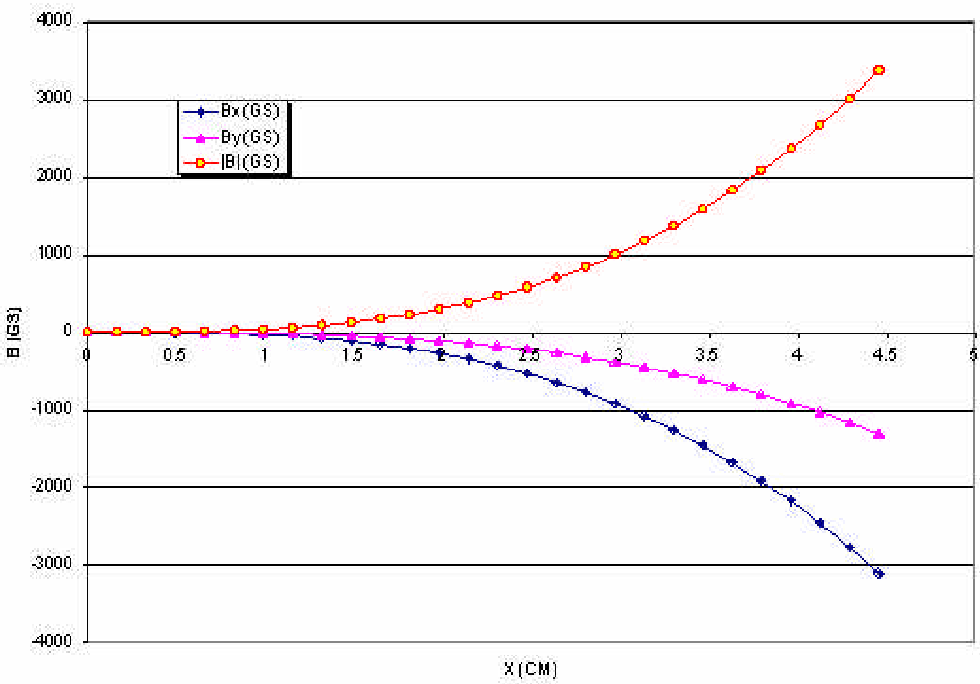

Figure 5: Magnetic field of the Octupole magnet along a 22.4 Degree line

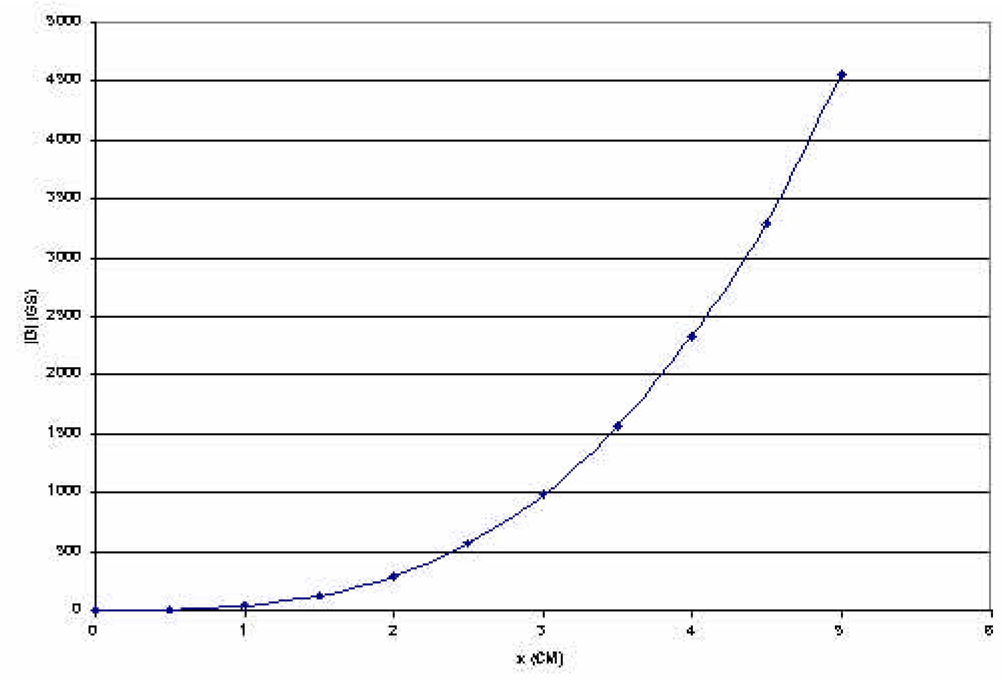

Figure 6: Magnetic field of the Octupole magnet along a 20 Degree line.

Figure 6 shows the magnetic field distribution along a 20 degree line. Figure 7 shows the field distribution along a 6.0176 degree line.

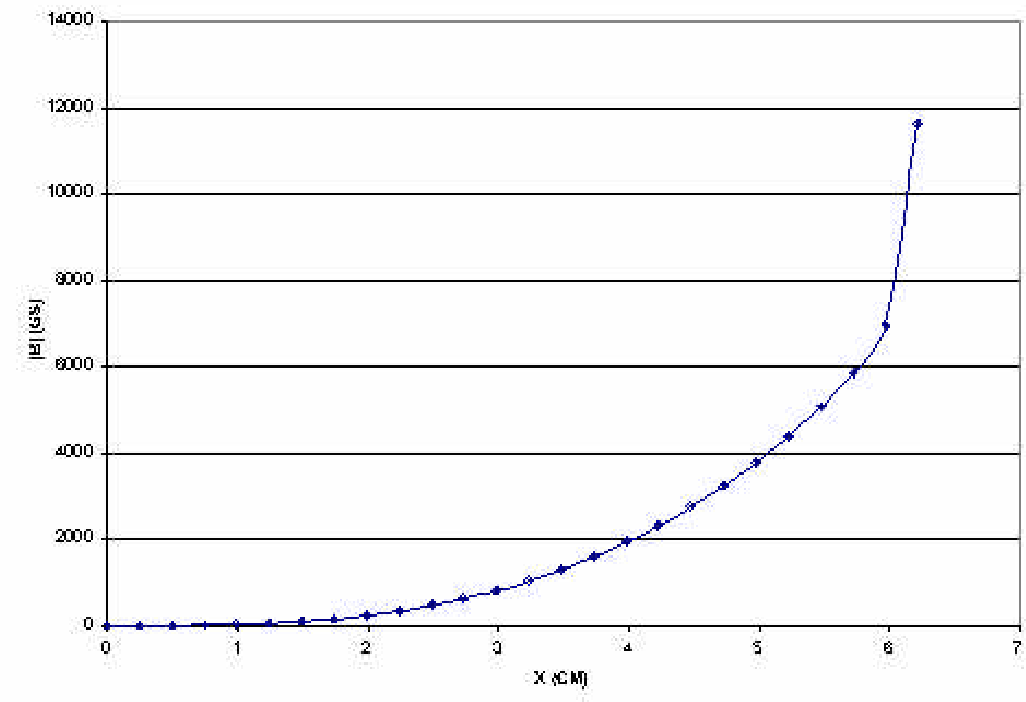

Figure 7: Magnetic field of the Octupole along a 6.0176 degree line.

2.2.3 Field distribution in the pole and yoke

Figure 8 shows the magnetic field components Bx and By inside the pole along a 22.4 degree line. Figure 9 shows the module of the magnetic field, |B|, along a 22.4 degree line. A maximum value of $B_{max} \approx 1.5$ T is reached.

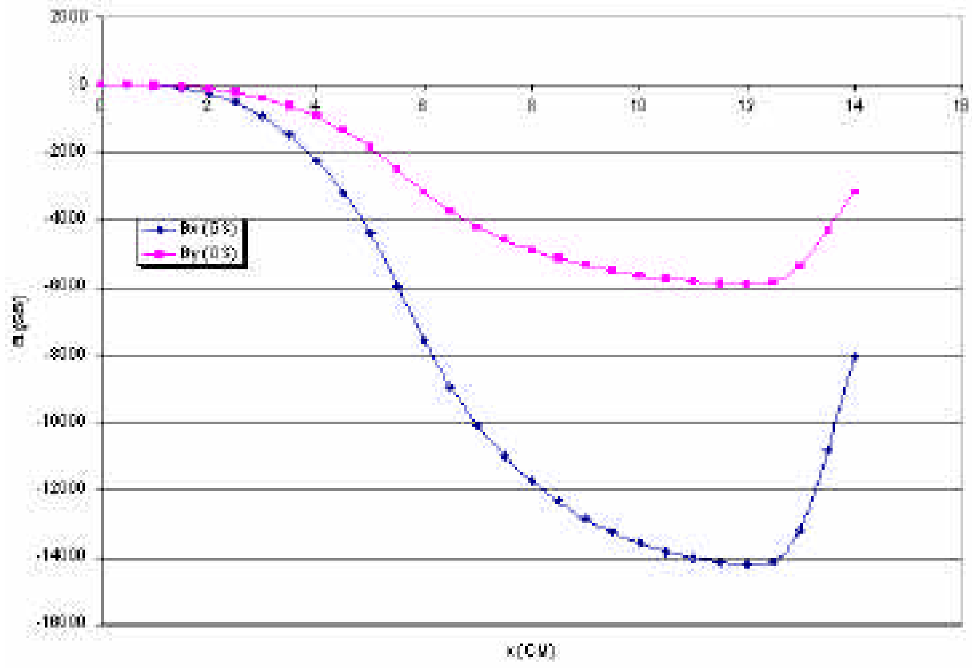

Figure 8: Magnetic field, Bx and By, of the Octupole along a 22.4 degree line.

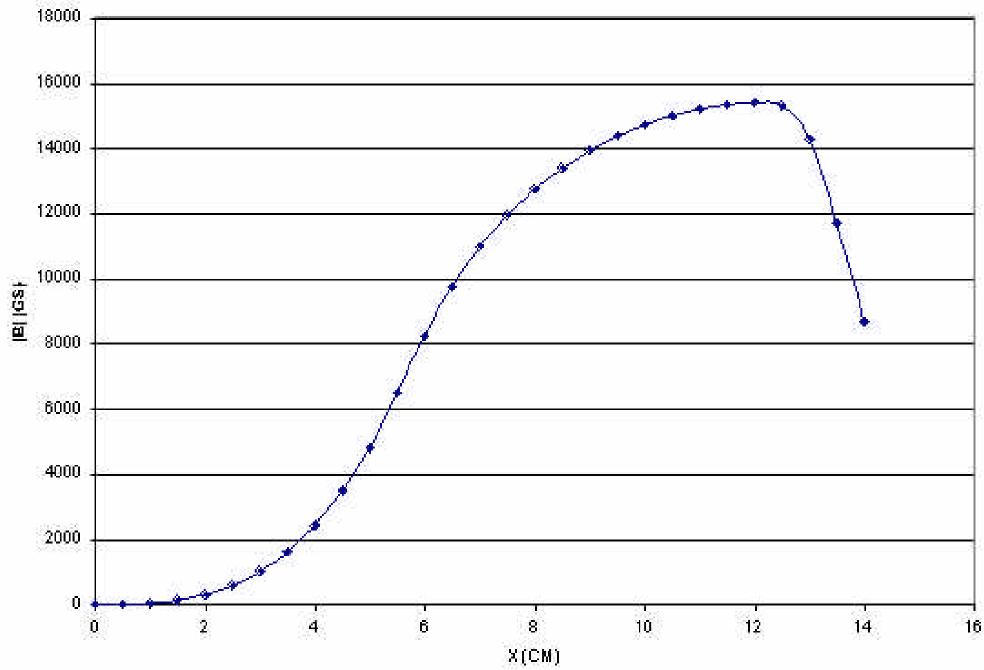

Figure 9: |B| distribution along a 22.4 degree line.

Figure 10 shows the components of the magnetic field Bx and By into the pole along a 20 degree line. Figure 11 shows the module of the magnetic field distribution, |B|, along a 20 degree line

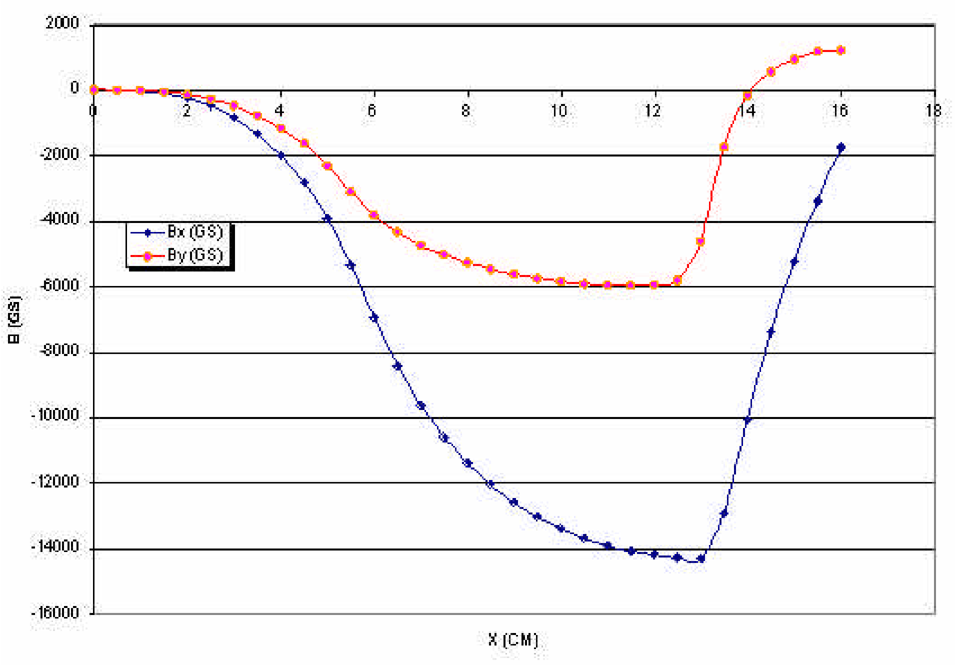

Figure 10: Bx and By magnetic field components of the Octupole along a 20 degree line

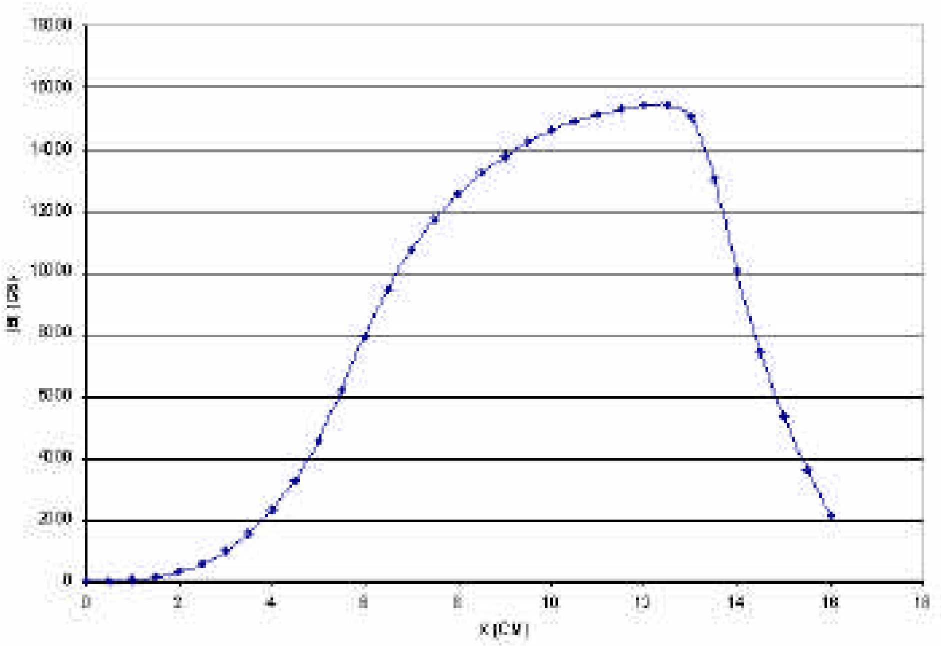

Figure 11: |B| magnetic field distribution along a 20 degree line.

2.2.4 Field distribution along the coil

Figure 12 shows the components of the magnetic field Bx and By through the coil along a 5 degree line. Figure 13 shows the module of the magnetic field |B| along a 5 degree line.

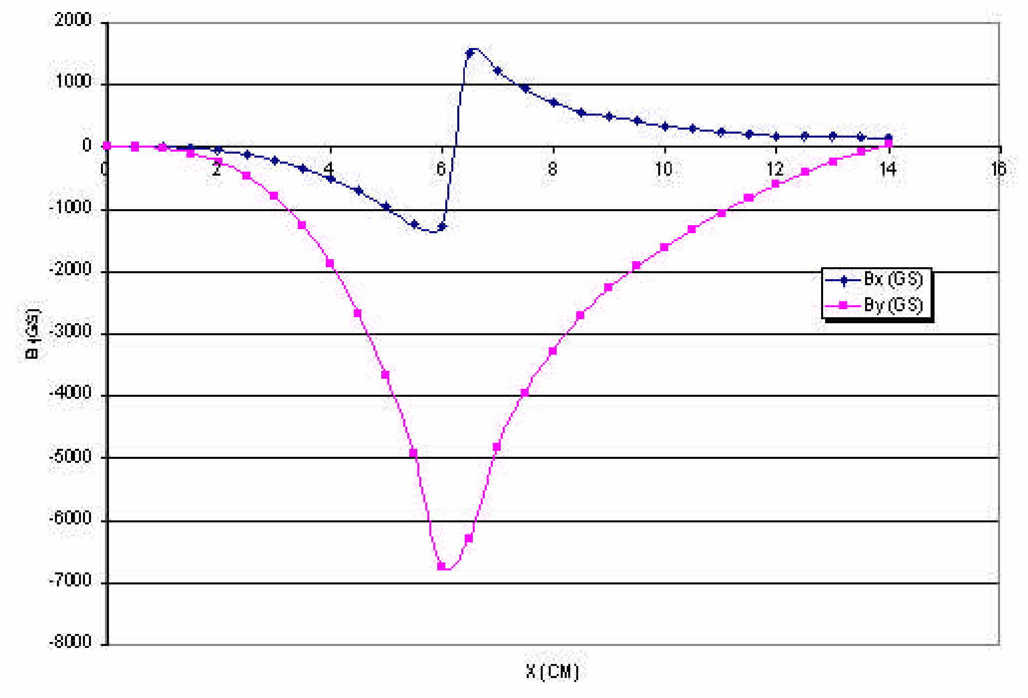

Figure 12 Bx and By magnetic field distribution along a 5 degree line.

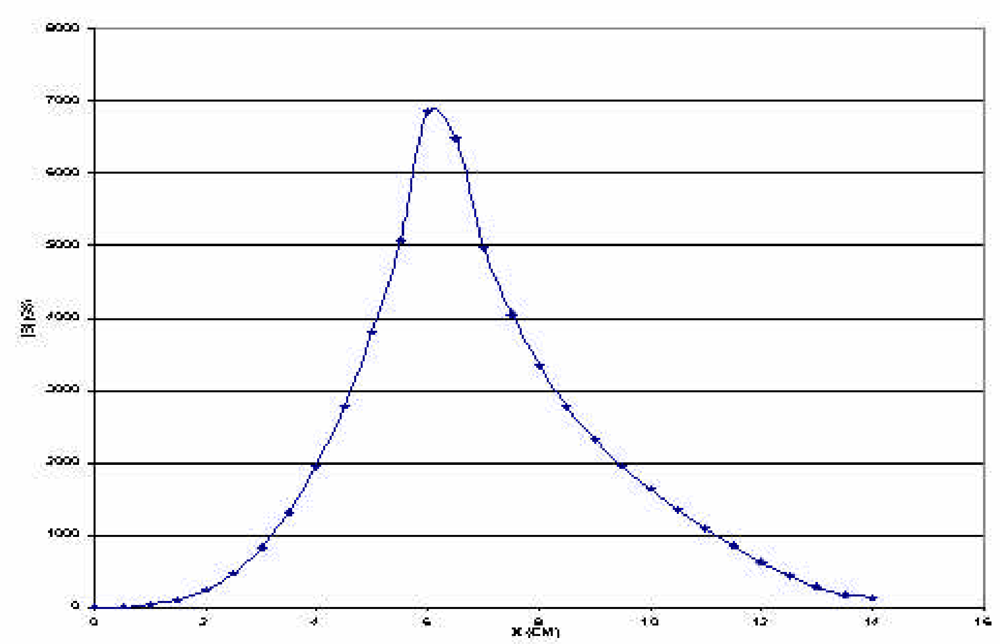

Figure 13: |B| magnetic field distribution along a 5 degree line.

2.3 Leak field

The magnetic field distribution with an external bounday extended to 28 cm from the iron yoke, to evaluate leak fields outside the octupole, has been simulated.
Figure14, showing the magnetic field distribution, indicates that all the magnetic flux is well contained inside the iron yoke.

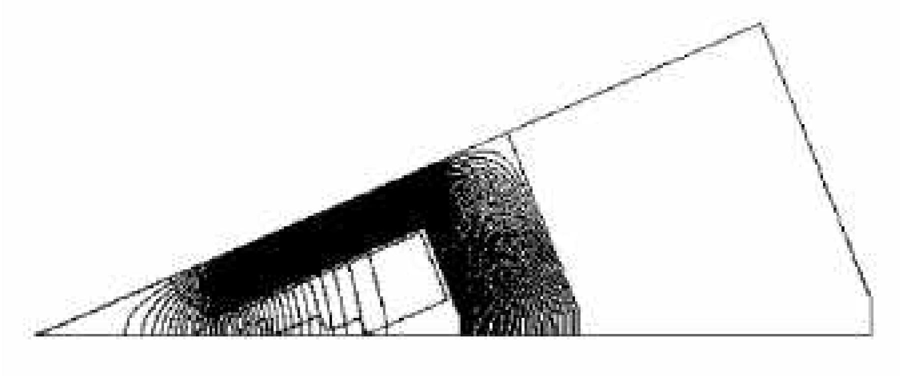

Fig.14 Magnetic flux distribution.

Figure 15 shows the magnetic field distribution along the x axis in the case of extended external boundary.

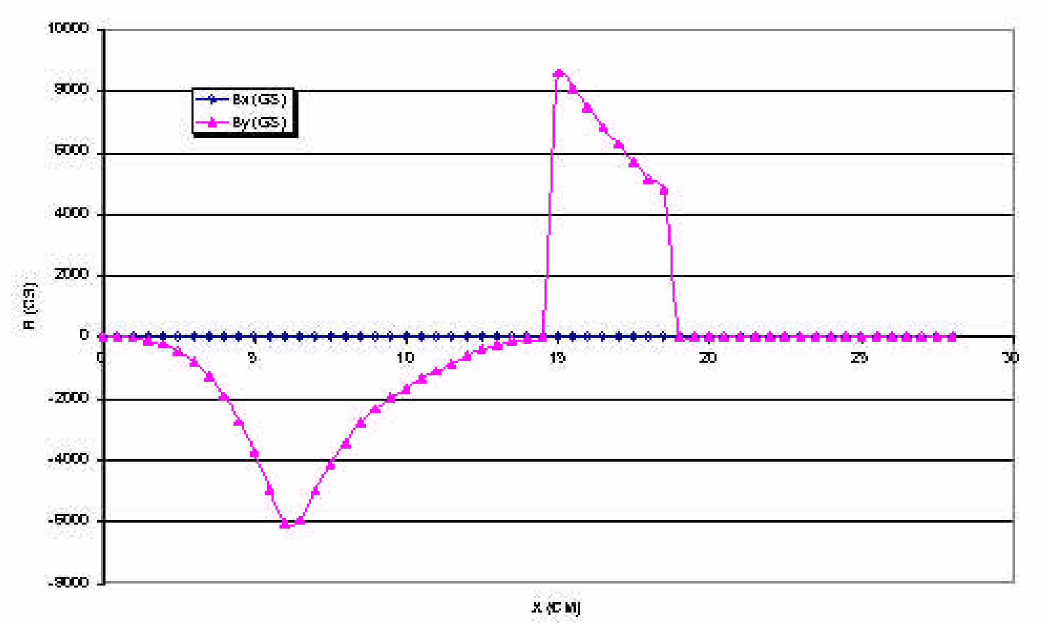

Figure 15: Magnetic field of the Octupole along the x axis extending 10 cm from the external boundary of the yoke.

Calculation shows that the field goes from 4800 Gauss to zero very sharply, without any leak field.