DAΦNE TECHNICAL NOTE

INFN - LNF, Accelerator Division

Frascati, Dec. 20, 1990

Note: **RF-2**

DOUBLE R.F. SYSTEM FOR DAΦNE

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A preliminary investigation of the main RF parameters was worked out in the Φ-Factory proposal LNF-90/031(R), based on a single RF system. It was shown that a 350 MHz RF system allows to obtain a 3 cm bunch-length in the anomalous bunch-lengthening regime with a reasonable peak voltage, provided the machine broad band impedance (BB) is less than 5 Ohms. Higher frequencies, in spite of the lower voltage and fewer cells, have the disadvantage of too low momentum acceptance, unacceptable from the Touschek life-time point of view. On the other side, lower frequencies require too high peak voltage; therefore a 350 MHz RF system was found to be a good compromise.

In this note we discuss the possibility of adopting a second, higher harmonic RF system to control the bunch-length with minimal voltage providing, at the same time, the necessary momentum acceptance. The higher harmonic cavity can be used either to increase or decrease the local voltage slope, which determines the equilibrium longitudinal size. We will explore the possibility of getting the design bunchlength of 3 cm in the anomalous bunchlengthening regime assuming a machine BB impedance of 2 Ohms. The main RF harmonic number is h = 120, the machine being envisaged to operate with a number of bunches up to 120. We will follow the formalism adopted in Refs. [1, 2, 3]

The double RF system provides a voltage:

$$V(\phi) = \widehat{V}\left[\sin(\phi+\phi_s) + k\sin(n\phi + n\phi_n)\right]$$

where ϕ is the longitudinal angular displacement, ϕ_s, (ϕ_n) is the synchronous phase relative to the main (harmonic) RF system, k is the higher harmonic peak voltage normalized to the voltage V of the main cavity, n is the harmonic order. By choosing $n\phi_n=-\pi$ the higher harmonic RF does not accelerate the bunch but changes the local time derivative around the synchronous phase by the factor (See Fig. 1):

$$\left(\dot{V}_o\right)_{double} = \left(\dot{V}_o\right)_{single}\left(1 + \frac{nk}{\left|\cos(\phi_s)\right|}\right)$$

yielding a bunchlength [4]:

$$(\sigma_l)_{double} = \frac{(\sigma_l)_{single}}{\sqrt{1 + \frac{nk}{|\cos(\phi_s)|}}}$$

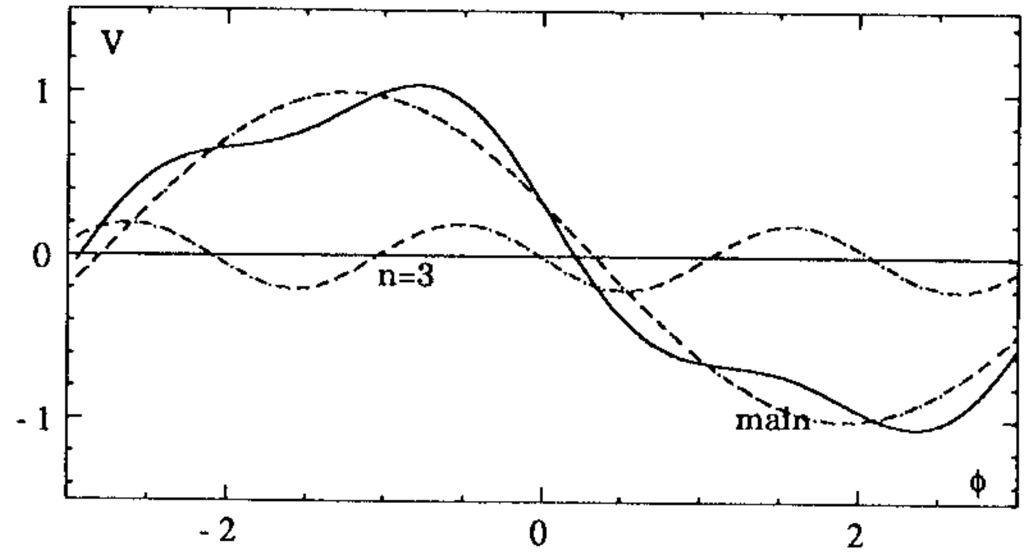

Fig. 1 - Voltages of the Double RF System.

The bunch shortens for positive k, while lengthens for negative one. The longitudinal motion may become unstable if $k < -\cos(\phi_s)/n$, for which the local voltage derivative becomes positive. The case of zero voltage slope is usually adopted when a maximum bunch-lengthening is desired. However, in our case, too high peak voltages would be required to keep the bunch at its nominal length of 3 cm, thus making this option unpractical.

In the anomalous bunch-lengthening regime, the bunch-length is computed taking into account the momentum spread induced by the microwave instabilities. With simple algebra we get:

$$nk = \frac{NecR^2}{\sqrt{2\pi}\, h\hat{V}\, \sigma_l^3} \left(\frac{Z}{p}\right)_o^{BB} - \cos(\phi_s)$$

where

$$\cos(\phi_s) = \sqrt{1 - \left(\frac{U_{loss}}{e\hat{V}}\right)}$$

which relates the higher harmonic cavity parameters (n, k) to the main RF (h,V) ones through the machine main radius R, the number of particles per bunch N, the equilibrium bunch-length and the machine BB impedance. It is worth reminding that the synchronous phase also depends on the machine BB impedance through the parasitic losses. Calculations carried out with ZAP code [5] give 8 KeV in a 2 Ω BB resonator with a bunch-lenght of 3 cm, pushing up the total loss (radiation + parasitic) to 22 KeV.

In Fig. 2 we give the curves of k versus the main RF cavity voltage at 380 MHz for a second (n = 2) and third (n = 3) harmonic. Dashed curves are drown assuming $\cos(\phi_S) = -1$.

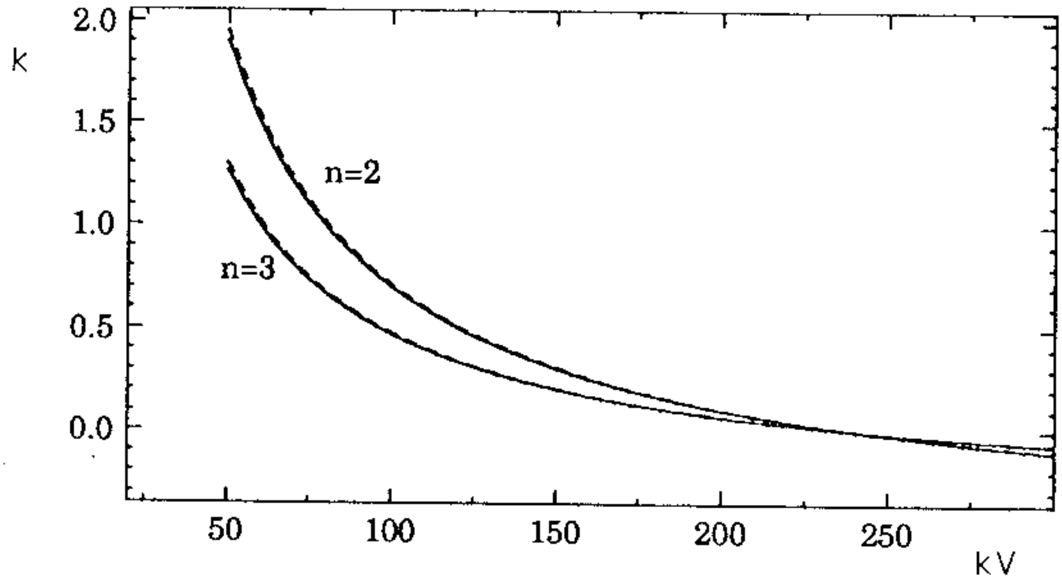

Fig. 2 - k versus V for n = 2, and n = 3.

We notice that σ_l=3cm bunch-length is obtained with a low voltage at the higher harmonics cavity (which does not supply energy to the beam), and this allows a reduction of the main RF system voltage. This might permit to optimize the number of cells of the RF system in terms of its contribution to the machine impedance. However, one cannot benefit of the lower voltage in the main RF cavity beyond the limit on the energy acceptance set by the Touschek life-time.

The maximum relative energy deviation accepted in the RF bucket, calculated with $\cos(\phi_S) = -1$, is (see Appendix 1):

$$\left(\frac{\Delta E}{E}\right)_{max} = \left[\left(\frac{2e\widehat{V}}{\pi\alpha_c hE}\right)\left(1 + \frac{k}{n}\right)\right]^{\frac{1}{2}}$$

In Fig. 3 we show the relative energy acceptance versus the main RF peak voltage for n = 2 and n = 3. We see that one can lower the main RF voltage at the expense of the energy acceptance. How much can we reduce V? This depends on the overall Touschek lifetime calculated including the effect of the physical and dynamical apertures. Should these effects be the main limits to the beam lifetime, it becomes useless to keep too high RF momentum acceptances.

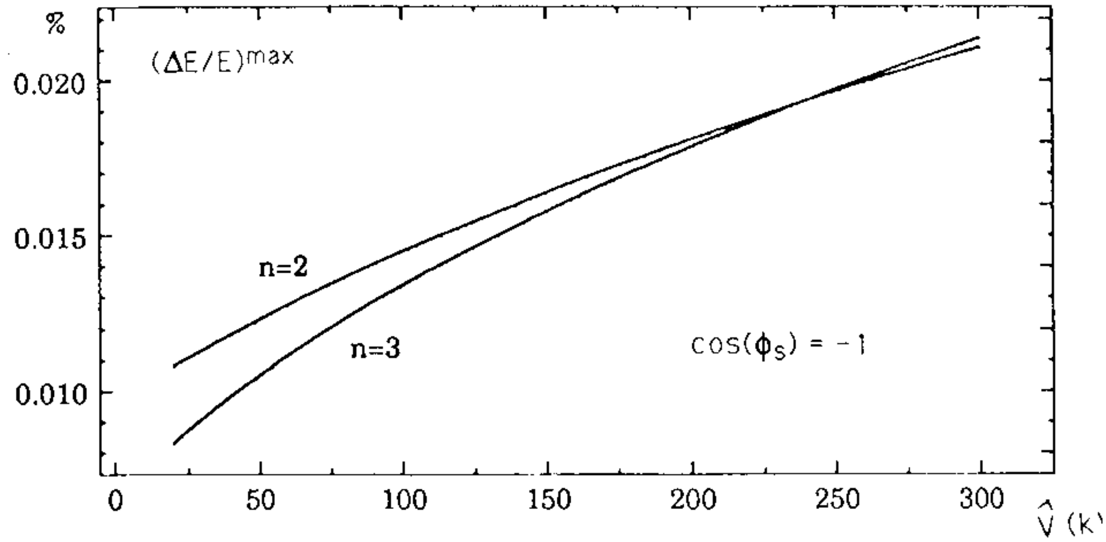

Fig. 3 - RF Momentum acceptance.

Finally it is worth noting that Eq. (6) fails for k>1; in fact in this range the higher harmonic voltage becomes dominant splitting the bucket in two (n=2) or three parts (n=3). This effect is clearly shown in Fig. 4 where the potential energy function of a third harmonic double RF system is shown. For $k \geq 1.2$, due to the potential well distortion, three buckets appear.

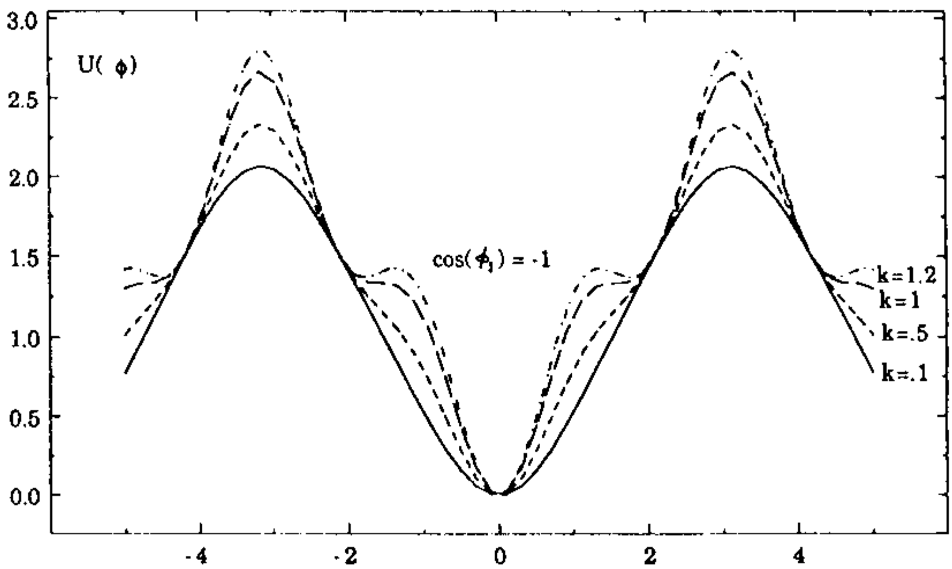

Fig. 4 - Potential energy function for a third harmonic double RF system.

The potential function behaves quadratically around the synchronous phase, therefore the bunch shape depends uniquely on the energy distribution which is gaussian only in the natural regime. As soon as collective effects occur, the bunch shape is modified by the potential well effect, which modifies the potential function, or by the microwave instabilities which affect the momentum distribution.

References

[1] P. Brahaman, A. Hofmann, P.B. Wilson, "A Higher Harmonic Cavity to Increase the Bunch Length in LEP-70", LEP/70-25, 1977.

[2] A. Hofmann, S. Myers, "Calculation on RF and Beam Parameters for Double RF System", LEP note 158, 1979.

[3] Y.H. Chin, "Double RF System for Bunch Shortening", ESG-108, LBL 29622, 1990.

[4] M.Sands, " The Physics of Electron Storage Rings", SLAC-121, 1970.

[5] J. Bisognano, S. Chattoparday, M. Zisman, Zap User's Manual, LBL Report No. LBL - 21270, 1986.

APPENDIX 1

The motion equation in the RF voltage neglecting the synchrotron damping is:

$$\ddot{\phi} = \frac{\alpha_c h \omega_o^2}{2\pi\,(E/e)} \left[V(\phi) - V_o \right]$$

Integrating the above equation yields:

$$\frac{\dot{\phi}^2}{2} - \frac{\alpha_c h \omega_o^2}{2\pi\,(E/e)} \int_o^{\phi} \left[V(\phi) - V_o \right] d\phi$$

the momentum acceptance is:

$$\left(\frac{\Delta p}{p}\right)_{RF} = \frac{\dot{\phi}^2_{MAX}}{\alpha_c h \omega_o} = \sqrt{\frac{e\hat{V}}{2\pi\alpha_c h\ E}} \left[\frac{1}{\hat{V}} \int_o^{\phi} \left[V(\phi) - V_o \right] d\phi \right]_{MAX}^{\frac{1}{2}}$$

Calling $U(\phi)$ the last expression in the square brackets we have:

$$U(\phi) = \cos(\phi_s)\left[1 - \cos(\phi)\right] + \sin(\phi_s)\left[\phi - \sin(\phi)\right] + \frac{k}{n}\left[1 - \cos(n\phi)\right]$$

In the approximation $\sin(\phi_S) = 0$, $\cos(\phi_S) = -1$ we get:

$$\left[U(\phi)\right]_{MAX}$$

$$\phi = +\pi \qquad\qquad \phi = +\tan^{-1}\sqrt{\frac{k-1}{k+1}}$$

Therefore for the cases of interest, i.e. for $k<1$, we have:

$$\left[U(\phi)\right]_{MAX} = 2\left[1 + \frac{k}{n}\right]$$