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SOLENOIDAL FIELD COMPENSATION

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ABSTRACT

Compensation of the effect of strong solenoidal magnetic fields on particle beams is studied. In e^+e^- Φ-factory the high detector magnetic fields are a strong perturbation of the optics due to the low beam energy, and the induced coupling is carefully corrected for luminosity optimization. A summary of different compensation methods is given, and the scheme adopted for DAΦNE, the Rotating Frame Method (RFM), is described.

1. Introduction

Electron-positron collider present designs are based in most cases on flat beams. The control of coupling between horizontal and vertical planes is one of the conditions to optimize luminosity, which is inversely proportional to the transverse section of the beams at the interaction point (IP).
The solenoidal field of the detector around the IP is the strongest source of coupling. Other coupling sources, distributed around the ring, such as quadrupole tilts, vertical closed orbit in sextupoles, longitudinal fringing fields,.., will not be here considered.
A solenoid rotates the normal transverse modes by the angle θ_R defined by the integral of the longitudinal magnetic field component along the closed orbit and inversely proportional to the particle energy:

$$\theta_R = \frac{1}{2B\rho} \int B_z(s)ds \qquad (1)$$

Compensation of the coupling eliminates this rotation and transforms the normal modes in horizontal and vertical ones.

2. Usual Compensation Methods

In 1978 Guignard developed the *resonance method*[1,2]: using Hamiltonian formalism and treating the coupling fields as a perturbation of the uncoupled optics he showed that four Skew Quadrupoles (SQ) on each side of a Detector Solenoid (SD) compensate the solenoidal perturbation outside the insertion and at its center (IP). The name of the method is due to the fact that the equations of motion are Fourier analysed and only terms near the sum and/or the difference resonance are considered. Assuming a thin lens model for the SQ, their strengths k_i and phase locations satisfy 4 linear equations:

$$\sum_i k_i \sqrt{\beta_{xi}\beta_{yi}} \sin\left[\mu_{xi} + \mu_{yi} - 2\pi\left(Q_x + Q_y - Q_+\right)\frac{z}{R}\right] = \theta_R\left(\sqrt{\frac{\beta_{xi}}{\beta_{yi}}} - \sqrt{\frac{\beta_{yi}}{\beta_{xi}}}\right) \tag{2}$$

$$\sum_i k_i \sqrt{\beta_{xi}\beta_{yi}} \sin\left[\mu_{xi} - \mu_{yi} - 2\pi\left(Q_x - Q_y - Q_-\right)\frac{z}{R}\right] = \theta_R\left(\sqrt{\frac{\beta_{xi}}{\beta_{yi}}} + \sqrt{\frac{\beta_{yi}}{\beta_{xi}}}\right) \tag{3}$$

$$\sum_i k_i \sqrt{\beta_{xi}\beta_{yi}} \cos\left[\mu_{xi} + \mu_{yi} - 2\pi\left(Q_x + Q_y - Q_+\right)\frac{z}{R}\right] = 0 \tag{4}$$

$$\sum_i k_i \sqrt{\beta_{xi}\beta_{yi}} \cos\left[\mu_{xi} - \mu_{yi} - 2\pi\left(Q_x - Q_y - Q_-\right)\frac{z}{R}\right] = 0 \tag{5}$$

Here the index 'i' refers to the SQ position, $Q_\pm$ is the nearest integer to $Q_x \pm Q_y$. The origin of the abscissa *z* is at the IP. The approximation of considering only terms near the resonance is the cause of the presence of global machine parameters in the equations.

The *perturbative method*[3,4], performed without introducing the harmonics, obtained the same result, except for the terms $2\pi\left(Q_x \pm Q_y - Q_\pm\right)z/R$ that disappeared.

Another method, which we call '*four knobs*, can find the SQ strengths just by imposing four constraints directly on the Half-Interaction Region matrix. Let's recall here few general properties of a matrix. The linear Jacobian representing the transverse transformation along any section of a ring can be written as a 4x4 matrix:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \tag{6}$$

where **A**, **B**, **C**, **D** are 2x2 matrices. The section is uncoupled if (x, x') do not depend on (y, y') and viceversa, that is if the two matrices **B** and **C** vanish, or the Jacobian is block diagonal.

Using the simplecticity condition:

$$\begin{pmatrix} \mathbf{A}^T & \mathbf{B}^T \\ \mathbf{C}^T & \mathbf{D}^T \end{pmatrix} \begin{pmatrix} \mathbf{J} & \mathbf{O} \\ \mathbf{O} & \mathbf{J} \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{J} & \mathbf{O} \\ \mathbf{O} & \mathbf{J} \end{pmatrix} \tag{7}$$

where $\mathbf{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, it is easy to show that this corresponds to 4 conditions, because the equations

$$\mathbf{B} = 0 \tag{8}$$

assure that also the matrix **C** vanishes. Since a 4x4 simplectic matrix has 10 degrees of freedom, once used four of them to decouple the matrix, 6 more parameters are available, and they are relative to the two diagonal matrices **A** and **D**.

Finding four elements whose strengths make the matrix off diagonal elements to vanish, is straightforward using any lattice design code, like MAD5) or NOLISY6), able to directly solve equations (8). The elements must introduce of course coupling terms, and can be therefore tilted quadrupoles or solenoids.
This direct and quick method was used for the first time to calculate the 4 SQ strengths for the L3 experiment at LEP7). The values came out to be very similar to those already computed with the resonant method8).

3. Rotating Frame Method (RFM)

The parameter which indicates the strength of the perturbation introduced by the magnetic field is the transverse plane rotation angle linked to the longitudinal field integral (Eq. 1). For LEP detectors this angle is about 2˚, while in DAΦNE9) it is about 45°, since the longitudinal magnetic field in the detectors are of the same order of magnitude. The four SQ scheme is not convenient in the DAΦNE IRs, essentially for a lack of space. In principle another scheme can be used: the rotation introduced by the SD is neutralized by two Compensating Solenoids (SC) placed on each side of the detector, with opposite magnetic field, to make the total integral of B_z along the particle trajectory vanish. An important inconvenient of this scheme is that the first low-β quadrupole has to be installed far away from the IP, so increasing chromaticity and aperture requirements.
The Rotating Frame Method (RFM) allows to insert quadrupoles between SD and SC, closer to the IP, without affecting the coupling correction. This method is based on the general properties of the solenoidal matrix: the matrix $\mathbf{M}_S$ representing a L long solenoid with uniform field B_z can be written as the product of two matrices **R** and **F** which commute:

$$\mathbf{M}_S = \mathbf{R}(\theta_R)\mathbf{F}(\theta_R) = \mathbf{F}(\theta_R)\mathbf{R}(\theta_R) \tag{9}$$

R represents a rotation by the angle θ_R:

$$\mathbf{R}(\theta_R) = \begin{pmatrix} \mathbf{I}\cos(\theta_R) & \mathbf{I}\sin(\theta_R) \\ -\mathbf{I}\sin(\theta_R) & \mathbf{I}\cos(\theta_R) \end{pmatrix} \qquad (10)$$

(**I** is the 2x2 unit matrix). Matrix **F** is block diagonal, equivalent to a quadrupole focusing on both planes with the same strength:

$$\mathbf{F} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{pmatrix} \qquad (11)$$

where **A** is the 2x2 matrix:

$$\mathbf{A}(L, \theta_R) = \begin{pmatrix} \cos\theta_R & \frac{L}{\theta_R}\sin\theta_R \\ -\frac{\theta_R}{L}\sin\theta_R & \cos\theta_R \end{pmatrix} \qquad (12)$$

A non uniform solenoid can be written as the product of small rectangular solenoids, each one defined by a length Δz and a rotation angle $\Delta\theta_R$. The total matrix can still be written as the product of a focusing matrix and a rotating one:

$$\mathbf{M}_S = \prod \Delta\mathbf{M}_S = \mathbf{R}(\theta_R) \prod \mathbf{A}(\Delta z, \Delta\theta_R) = \mathbf{R}(\theta_R)\, \mathbf{F}_S \qquad (13)$$

It is worth to point out that the rotation angle is the sum of the small rotations while the focusing strength is obtained with the product of the focusing matrices.
Let's consider the half-IR matrix $\mathbf{M}_H$ from the IP to the end of the SC:

$$\mathbf{M}_H = \mathbf{F}_C\, \mathbf{R}(-\theta_R)\, \mathbf{R}(\theta_R)\, \mathbf{F}_D = \mathbf{F}_C\, \mathbf{F}_D \qquad (14)$$

Subscripts ‘C’ and ‘D’ stand respectively for Compensator and Detector. This matrix is of course block diagonal.
Let's now add a quadrupole, represented by the matrix **Q**, between the detector and the compensator, with two drifts L_1 and L_2 in between as in Fig. 1:

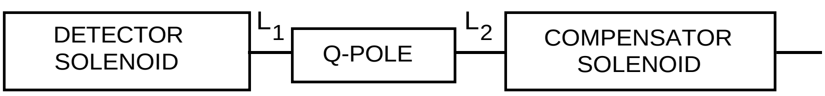

Figure 1: *Example of half Interaction Region Scheme*

The matrix from the IP to the SC entrance will be:

$$\mathbf{M}_Q = \mathbf{L}_2\ \mathbf{Q}\ \mathbf{L}_1\ \mathbf{R}(\theta_R)\ \mathbf{F}_D \tag{15}$$

$\mathbf{L_1}$ and $\mathbf{L_2}$ are multiple of the unit matrix **I**, and commute with **R**:

$$\mathbf{M}_Q = \mathbf{L}_2\ \mathbf{Q}\ \mathbf{R}(\theta_R)\ \mathbf{L}_1\ \mathbf{F}_D \tag{16}$$

If the quadrupole is tilted by the angle θ_R its matrix **Q** becomes:

$$\mathbf{Q}_R = \mathbf{R}(\theta_R)\ \mathbf{Q}\ \mathbf{R}(-\theta_R) \tag{17}$$

Using again the commutation property we get:

$$\mathbf{M}_{QR} = \mathbf{L}_2\ \mathbf{R}(\theta_R)\ \mathbf{Q}\ \mathbf{R}(-\theta_R)\ \mathbf{R}(\theta_R)\ \mathbf{L}_1\ \mathbf{F}_D = \mathbf{R}(\theta_R)\ \mathbf{L}_2\ \mathbf{Q}\ \mathbf{L}_1\ \mathbf{F}_D \tag{18}$$

and the half-IR matrix is:

$$\mathbf{M}_H = \mathbf{M}_C\ \mathbf{M}_{QR} = \mathbf{F}_C\ \mathbf{R}(-\theta_R)\ \mathbf{R}(\theta_R)\ \mathbf{L}_2\ \mathbf{Q}\ \mathbf{L}_1\ \mathbf{F}_D = \mathbf{F}_C\ \mathbf{L}_2\ \mathbf{Q}\ \mathbf{L}_1\ \mathbf{F}_D \tag{19}$$

$\mathbf{M}_H$, product of block diagonal matrices, is block diagonal: a quadrupole placed in between the detector and the compensator does not introduce coupling, provided it is tilted by the same angle as the transverse coordinate frame. This holds of course for any number of quadrupoles between the SD and the SC.

4. Application of RFM to DAΦNE

The RFM has been adopted in the DAΦNE IRs design, for the compensation of the two high field detector solenoids, KLOE and FINUDA, where the low-β quadrupoles are installed inside the detectors (see Fig. 2 for a sketch of KLOE IR).

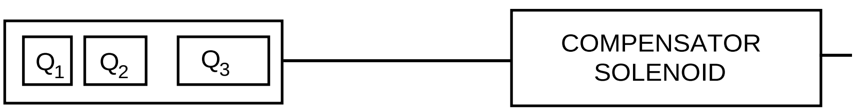

Figure 2: *KLOE IR*

Exact application of the RFM implies that each quadrupole immersed in the solenoidal field should be continuously rotated as an helix. This is not pursuable because, apart from technological difficulties, the rigidity of the scheme would ask for very strict tolerances on detector fields and beam energy. Hence, quadrupoles are tilted as a whole by the angle θ_R corresponding to their longitudinal midpoint. The half IR matrix obtained exhibits a small residual coupling which can be corrected by applying Eqs. (8) and choosing as knobs three independent supplementary rotations of the low-β quadrupoles, $\delta\theta_{Ri,}$ and a correction of the compensator field, resulting in a $\delta\theta_C$. The example for the KLOE detector is shown in Table 1. Being the angle an odd function with respect to the IP, each quadrupole on the left of the IP is rotated by the same and opposite angle of its symmetric quadrupole on the right.

Table 1: *KLOE IR*

KLOE	θ_R (˚)	$\delta\theta$ (˚)
Q1	**+5.58**	**+0.29**
Q2	**+10.28**	**-0.02**
Q3	**+15.20**	**-0.27**
Compensator	**-21.22**	**+0.34**

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